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Stochastic space-time inversion schemes for NEXRAD precipitation

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Abstract. An inversion scheme is applied to downscale daily precipitation observed at a spatial scale corresponding to a pixel size of 256x256 *km* into a spatial scale corresponding to a pixel size of 2x2 *km*. The inversion algorithm is a composite scheme consisting of 2 sub-models, SSTDM (Stochastic Space-Time Disaggregation Model) which considers hierarchical structure of the spatial and temporal statistical dependencies of precipitation and IRCM (Intermittent Random Cascade Model) which is based on scale invariant feature and reproduces self-similarity structure and spatial intermittent cluster formation. For SSTDM, Valencia and Schaake's general disaggregation model (1973), which was originally developed for multi-site, multi-season streamflow disaggregation, was modified in this work for use in the downscaling of grid precipitation. IRCM adopted the mass re-distribution concept of the multiplicative random cascade structure (Kang and Ramirez, 2001, Over and Gupta, 1996) The results of applying this inversion scheme on the precipitation field of July, 1997 are presented and discussed. This model is expected to be suitable for downscaling of GCM model output to drive hydrologic models and carry out assessments of the impacts of climate variability on hydrology and water resources.

1. Introduction

Because of scale differences between global (or regional) climate or atmospheric models and regional or local hydrologic models, rainfall downscaling is required for coupling of those models. This coupling is necessary, for example, in order to evaluate the local and regional hydrologic effects of short and long term climatic variability. Therefore, a methodology must be devised to scale down hydroclimatic information from numerical global- and meso-scale atmospheric models or from observations.

In *statistical downscaling* schemes, sub-grid temporal and spatial scale details of climatic variability, in particular precipitation, are obtained by using statistical climate inversion techniques in which the statistical characteristics of the spatial and temporal variability of hydroclimatic fields is preserved as a function of scale. Statistical techniques are commonly based on methods from regression (linear or non-linear), stochastic process, nonlinear dynamics, artificial neural networks, Markov processes, multiplicative random cascade models, etc. Each methodology has its unique strengths for reproducing specific statistical features of the fields. Therefore, the modeling process requires a definition of the most efficient combination of methodologies required to achieve reproduction of the desired statistical properties of the underlying data set. Generally, using stochastic models is required when a data set shows strong spatial and temporal dependency. In addition, multiplicative random cascade models have the capability of reproducing the scale invariance features, the clustering, and intermittency that are characteristic of precipitation fields in space and time, while at the same time imposing relatively modest computational burden. This paper introduces an inversion scheme that is applied to invert daily precipitation observed at a spatial scale corresponding to a pixel size of 256x256 *km* into a spatial scale corresponding to a pixel size of 2x2 *km*. The inversion scheme is a composite scheme consisting of a stochastic disaggregation model and of a multiplicative random cascade model.

2. Data description

The Global Hydrology Resource Center (GHRC) generates 2x2 *km* daily accumulated precipitation data for the continental United States (CONUS) based upon composites of radar reflectivity data received from WSI (Weather Services International) corporation. The WSI daily radar reflectivity data is in image form with a size of 1887 rows by 3661 columns and has a resolution of 2x2 *km*. Its coverage of the original scan for continental U.S. extends from latitude 20 N to 53 N and longitude 130 W to 60 W. This 15-minute radar data are converted into a composite daily rainfall total, which has the unit of *in/day* and is eventually organized into 13 classes of rainfall accumulations. In this study, the data for central U.S. in July 1997 was clipped out of the original data. The clipped data (1024x1024 *km*, 512x512 pixels) ranges from 33.8 N to 43.0 N and longitude 105.7W to 95.9 W (from east of Rocky mountain to the lower Missouri and Arkansas river basin). The study region encompasses the Front Range and plains area

The radar precipitation data used for this study correspond to a field of 512x512 pixels at its highest resolution. This high-resolution field is averaged by using a branching number of 4, so that a series of rainfall fields, each at a consecutively lower resolution is obtained spanning the entire range of scales. Kumar and Foufoula-Georgiou (1993a) pointed out that the small-scale fluctuation (high frequencies and short wave length) exhibit self-similarity and is dominated by the microscopic turbulent behavior. However, at large scale this behavior breaks down to accommodate the effects of synoptic climatic conditions affecting the particular rain-producing mechanism. Kumar and Foufoula-Georgiou's (1993b) experiment shows that the squall line rainfall fluctuations exhibited self-similar characteristics up to a scale of *25-30 km*. The temporal correlogram of NEXRAD dataset used in this research shows 32 *km* as the critical scale, consistent with the result of Kumar and Foufoula-Georgiou (1993b).

3. Stochastic space-time downscaling model

The model consists of 2 sub-models: Stochastic Space-Time Disaggregation Model (SSTDM) and Intermittent Random Cascade Model (IRCM). SSTDM is used for spatial scales above 32 *km* to reproduce the observed correlation structure. IRCM is used for scales below 32 km scale to reproduce the self-similarity and spatial intermittent cluster formation. Figure 1 illustrates the model structure.

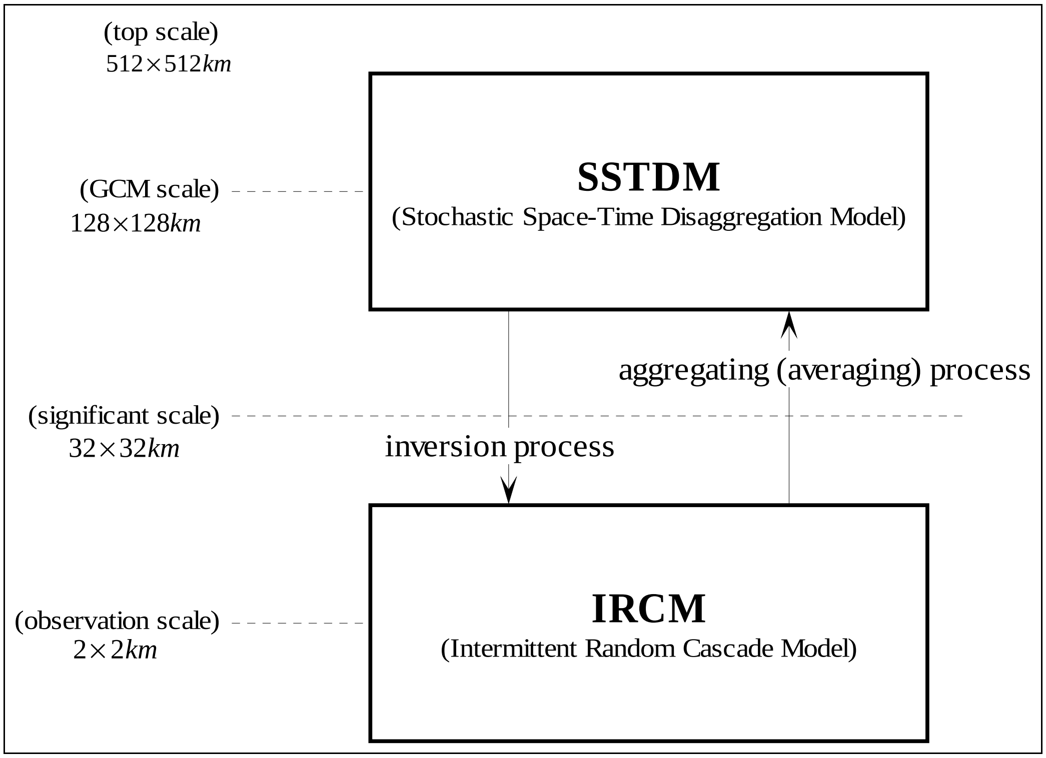


Figure 1 Integrated modeling process

3.1 Stochastic model for grid precipitation

Valencia and Schaake's general disaggregation model (1973), which was originally developed for multi-site, multi-season streamflow disaggregation, was modified in this work for use in the downscaling of grid precipitation. The basic mathematical form of the space-time disaggregation process, *Z(t)*, for grid precipitation is

$$\mathbf{Z(t)} = \mathbf{AX(t)} + \mathbf{BW(t)} \qquad (1)$$

where X and Z are log-transformed precipitation variables, which have zero, mean and unit variance, A, B are coefficient matrices, W is a vector of correlated random variables, which is independent of X. The rainfall was transformed to a logarithmic standardized dimensionless variable to ensure positiveness of the matrix components. The covariance matrices consist of spatial covariances of 1-pixel spatial distance and lag-1 (day) temporal covariances and were constructed under the homogeneity and isotropy assumptions. In addition, the fluctuation of normalized rainfall is assumed to be independent of the rainfall intensity. Figure 2 illustrates the stochastic space-time disaggregation process.

3.2 Intermittency random cascade model

The self-similar and spatially clustered structure of precipitation fields is described by the intermittency random cascade model. This model is applied to scales corresponding to pixel sizes of 32 *km* and smaller and daily time scale. The random cascade model neglects temporal correlation.

This multiplicative process proceeds through all cascade levels, so that the mass re-distribution of this model in cube Δ_n^i of n^{th} cascade level is expressed as (Kang and Ramirez, 2001, Over and Gupta, 1996):

$$r_m\left(\Delta_n^i\right) = R_0 \prod\nolimits_{j=1}^{m} W_j(i), \text{for } i = 1, 2, \ldots, b^n \qquad (2)$$

where $r_m\left(\Delta_n^i\right)$ is the precipitation of m^{th} cascade level in the cube Δ_n^i, *m* is the cascade level of high frequency disaggregation and *n* is the cascade level of low frequency disaggregation. *W* is a random cascade generator.

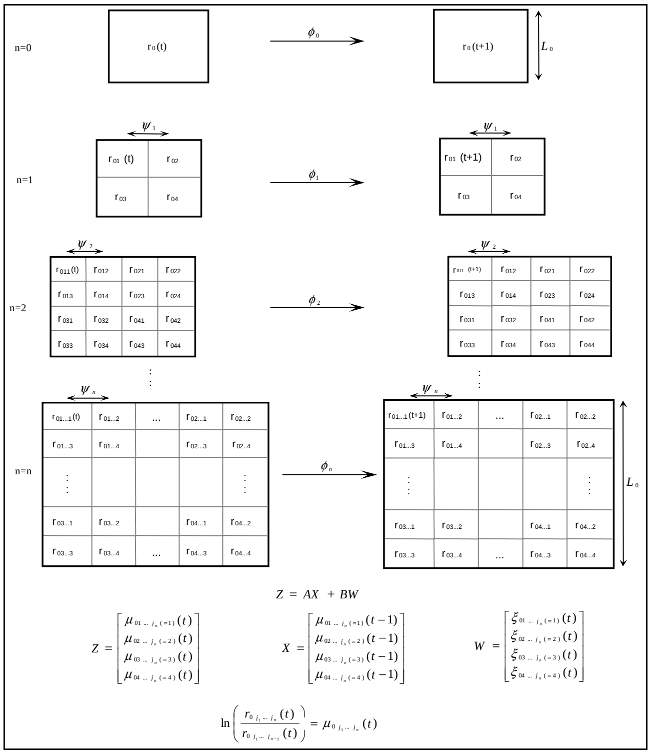


Figure 2 Schematic of the stochastic space-time disaggregation scheme

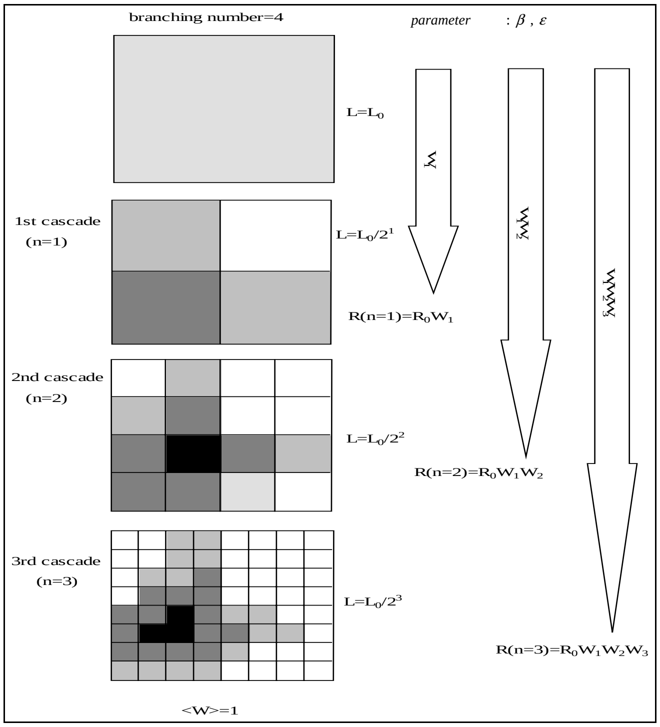
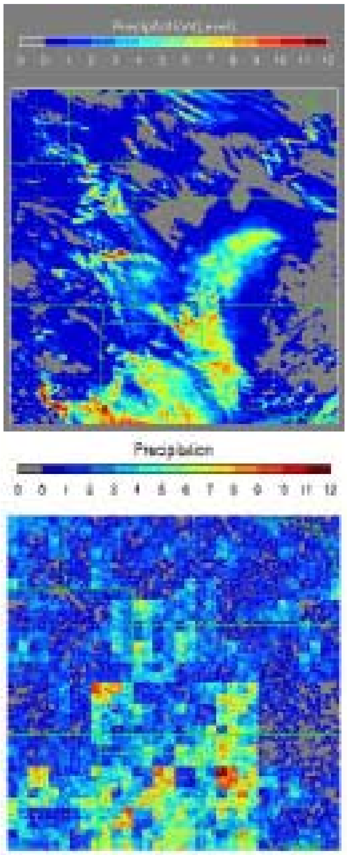


Figure 3 Schematic diagram of intermittent random cascade model

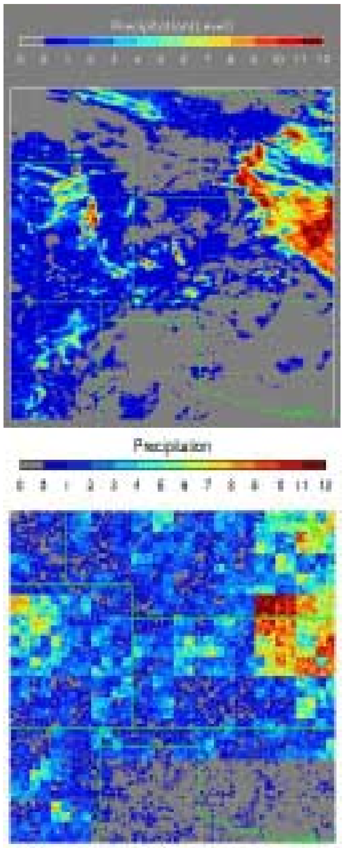
The cascade generator, *W*, is expressed by the multiplication of an *intermittency generator, B*, which reproduces the spatial clustering of observed precipitation and a *positive generator*, *Y*, which is a positive log-normally distributed random variable. The cascade structure of random cascade model is demonstrated in Figure 3.

4. Results

This model was tested on the precipitation field of July 1997. The NEXRAD observation and simulated fields are compared in Figure 4. The spatial correlation of the simulated precipitation fields reproduces well the patterns of the observations under the assumption of stationarity (Figure 5). The lag-1 temporal correlation of the simulation as a function of scale, as shown in Figure 6, reproduces the observed behavior adequately. However, the intermittency parameter, β, appears to be underestimated by the model (see Figure 7). However, if we consider the lowest class of NEXRAD is just *0.05 in/day* and that it occupies the largest portion of the precipitation field, this underestimation does not have critical effect on the overall behavior of the underlying field. Based on these excellent preliminary results of the proposed inversion scheme, it is clear that this model will be suitable for downscaling of GCM model output to drive hydrologic models and carry out assessments of the impacts of climate variability on hydrology and water resources.

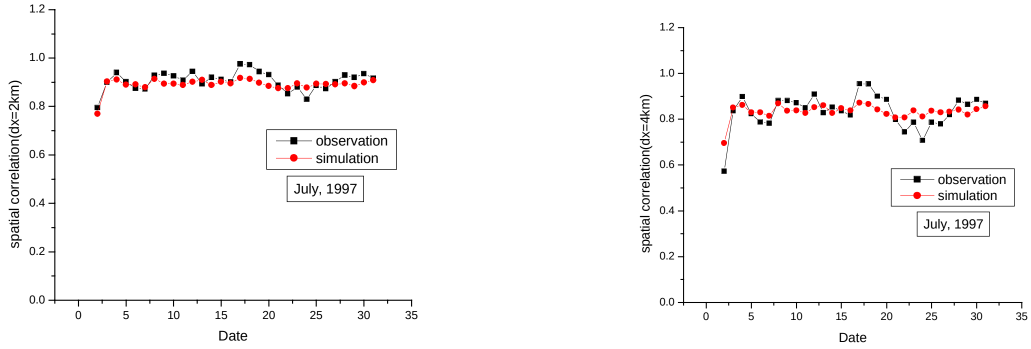


(a) Space-time stochastic rainfall model (Jul. 6., 1997)



(b) Space-time stochastic rainfall model (Jul. 8., 1997)

Figure 4 Comparison of NEXRAD and simulated precipitation field (Jul. 6 and 8 1997)



(a) scale=2*2km (b) scale=4*4km

Figure 5 Comparison of Spatial correlations for fields of 2 different scales

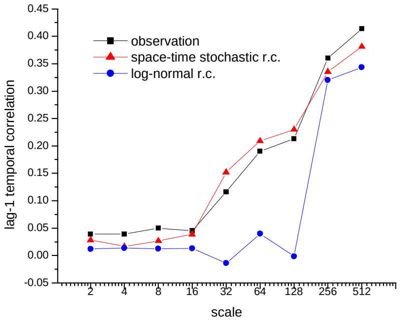


Figure 6 Comparison of Lag-1 temporal correlation

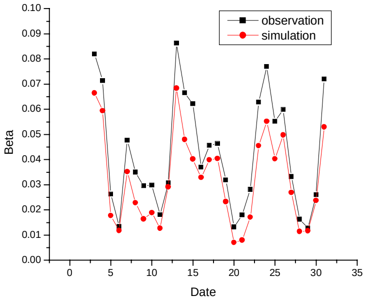


Figure 7 Comparison of self-similarity parameter, β

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