A fast forward-inverse observation operator for data assimilation of GPS refractivity profiles

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Overview

• Principle of forward-inverse refractivity mapping

• Simulations with a model weather front
  - evaluation on geometric height levels
  - evaluation on pressure levels
  - approximately taking into account the ray bending

• A new refractivity observation operator

• “Truth” & errors in a new perspective

• Conclusion—what should be assimilated?
Mimicking the observations and the Abel inversion using finite straight lines.

Basic requirement: \[
\int_{-L/2}^{L/2} N(x, y) \, dx = \int_{-L/2}^{L/2} N_{\text{map}}(r) \, dx
\]

- Discretized and solved for \( N_{\text{map}}(r) \)
- \( N(x, y) \) evaluated at (pressure) levels of NWP model
Weights for the mapping interpolated to a regular grid in altitude-latitude coordinates (tangent point at 5 km)

Mapping not restricted to fixed 2D plane; weights are in practice centered at tangent points (drifting during the occultation)

- In practice mapping consists of fast recursion formula
- Exact in (hypothetical) case of spherical symmetry
- Can be applied to model pressure levels instead of height

How well does it represent measurements with horizontal gradients?
Simulations with model of a weather front
Eight separate simulations with different positions and orientations of the front relative to the rays

- Simulated occultation observations via accurate 3D ray-tracing
- Retrieved refractivity using spherical symmetry (Abel transform)
- Compared with the fast forward-inverse refractivity mapping
• Retrieved profiles differ from each other by a few percent

• Tangent point profiles differ from retrieved profiles by a few percent
• Representativeness of mapping on height levels better than 0.5%
• Mapping on pressure levels only degrades results slightly (biased?)
Taking into account the ray bending

\[ \frac{d\alpha}{d\theta} \approx 1 - \frac{\theta_1^2}{\tilde{\theta}_1^2} \]

\[ \frac{d\alpha}{d\theta} \approx -R_e \frac{dN_{ret}}{dr} \]

**Trick:** Evaluate refractivity at slightly larger angles \( \tilde{\theta} \)

- Mapping-weights are not altered (mapping remains linear)
Mapping results with bending

- Appreciable improvement for mapping on height levels
- Small improvement for mapping on pressure levels
Example of observation operator

1. Horizontal interpolation (along pressure surfaces) of the temperature and specific humidity to the points used in the mapping

2. Evaluation of the refractivity at these points

3. Mapping the refractivity into a profile at the tangent points using the mapping operator

4. Integration of the hydrostatic equation to obtain a precise relation between pressure and geometric height at grid points near the tangent points

5. Horizontal interpolation of the geometric height to the tangent point locations
**Forward-inverse principle in general**

**Strength:** Near cancellation of otherwise crude approximations ⇒ fast but still reasonably accurate

- Useful for all kinds of occultation measurements (absorption too)
- Could perhaps even be adapted for assimilation of radiances, etc…
Advantages of refractivity mapping

- The mapping is linear, i.e., the weights used in the mapping do not depend on the atmospheric variables. The weights and the points where to evaluate the refractivity need only to be calculated once and can be re-used for all iterations in the assimilation process.

- Because the mapping is based on finite integrations in both a forward and inverse part, extrapolation of the NWP model above its highest level is not necessary.

- The mapping can be applied directly to pressure levels, thereby avoiding integration of the hydrostatic equation for a large number of locations. Hydrostatic integration is only essential at grid points near the tangent points.
Truth and consequences (errors)

Definition of “truth”: The horizontally homogeneous refractivity which would reproduce the actual phase observations (disregarding ionospheric effects and observation errors)

- Observation/retrieval errors
  - Orbit errors
  - Receiver related errors
  - Anti-spoofing
  - Local multipath
  - Ionosphere correction residual
  - Ellipsoidal earth correction residual
  - Upper boundary condition (a priori)
  - Surface reflections
  - CT/FSI approximations

- Representativeness errors
  - Model interpolation errors
  - Uncertainty in refractivity formula
  - Forward-inverse mapping errors
  - Model hydrostatic integration error

- Super refraction ambiguity?

Which of these are the most important?
Dominating errors in the troposphere

• Tracking errors (∼1–10%) [Beyerle et al., 2003; Ao et al., 2003]
  
  Bias! Will it be completely solved with open loop?

• Thermal noise, ionosphere (< 0.01%) [Kursinski et al., 1997]

• Anti-spoofing noise (?)

• CT/FSI approximations (?) — random error propagation (?)

• Forward-inverse mapping errors (∼0.2%) [This workshop]

• Other representativeness errors (?)

Refraction error covariances needed
Conclusion—what should we assimilate?

- Linear refractivity mapping is fast (how fast?) and reasonably accurate.

- Simple (tangent point) refractivity profile assimilation would be faster but less accurate (∼ factor of five).

- Bending angle assimilation is potentially the most accurate but too slow ⇒ trade-off between accuracy and speed ⇒ corresponding error probably not smaller than linear refractivity mapping ⇒ good argument for operational assimilation of bending angles seems hard to find.