Retrieval of electron density profiles at CDAAC

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Topics of Lecture

- Ionospheric refractive index
- Carrier phase observables
- Ionospheric linear combination of phases
- Principle of retrieval technique
- Upper boundary retrieval
- Examples & Applications
Characteristic frequencies related to ionospheric radio propagation:

- electron angular *plasma frequency*: \( \omega_p^2 = e^2 N_e / (m \varepsilon_0) \)
- electron angular *gyro frequency*: \( \omega_c = e B / m \)
- effective electron *collision frequency*: \( \nu_e \)

Short-hand notations:

\[
X = \frac{\omega_p^2}{\omega^2}, \quad Y = \frac{\omega_c}{\omega}, \quad Z = \frac{\nu_e}{\omega}, \quad U = 1 - iZ
\]

Neglecting massive positive ions (permissible for radio waves with \( \omega \gg 1 \text{kHz} \)), the dispersion relation depends upon \( X, Y, U(Z) \), and the angle \( \theta \) between the *geomagnetic field vector* and the wave *normal* (i.e., the ionosphere is generally *anisotropic*)
The Appleton-Hartree (or Appleton-Lassen) formula:

\[
n^2 = 1 - \frac{X(U - X)}{U(U - X) - \frac{1}{2}Y^2 \sin^2 \theta \pm \sqrt{\frac{1}{4}Y^4 \sin^4 \theta + Y^2 \cos^2 \theta (U - X)^2}}
\]

- two solutions (±) for waves propagating in the ionosphere
- plus sign: ordinary wave
- minus sign: extraordinary wave

GPS signals are mainly right-hand circularly polarized, giving rise to either ordinary waves (southern geomagnetic hemisphere) or extraordinary waves (northern geomagnetic hemisphere)
Ionospheric refractive index

Series expansion:

\[ n \approx 1 - \frac{1}{2} X \pm \frac{1}{2} XY |\cos \theta| - \frac{1}{8} X^2 - \frac{1}{4} XY^2(1 + \cos^2 \theta) - i\frac{1}{2} X Z \]

At GPS frequencies the order of magnitude of the terms are:

\[ 1 \approx 10^{-4} \quad \pm \quad 10^{-7} \quad - \quad 10^{-9} \quad - \quad 10^{-10} \quad - \quad i10^{-9} \]

• Four last terms are ignored in ionospheric retrievals

Ionospheric refractive index at GPS frequencies:

\[ n \approx 1 - \frac{1}{2} X = 1 - \frac{1}{2} \frac{\omega^2}{\omega_p^2} = 1 - \frac{C}{f^2 N_e} \]

where \( C = \frac{e^2}{8\pi^2 m \varepsilon_0} \approx 40.3 \text{ m}^3/\text{s}^2 \)
Carrier phase observables

Idealized phase path observation equations in the ionosphere (disregarding satellite clock errors, special and general relativistic terms, phase ambiguity bias, random noise, . . . etc):

\[
L_1 = \int_{\text{GPS}}^{\text{LEO}} n_1 dl_1 \approx \int_{\text{GPS}}^{\text{LEO}} (1 - \frac{C}{f_1^2} N_e) dl_1, \quad (f_1 = 1.57542 \text{ GHz})
\]

\[
L_2 = \int_{\text{GPS}}^{\text{LEO}} n_2 dl_2 \approx \int_{\text{GPS}}^{\text{LEO}} (1 - \frac{C}{f_2^2} N_e) dl_2, \quad (f_2 = 1.22760 \text{ GHz})
\]

- Clock errors and relativity terms are the same for the two signals
- Phase ambiguities irrelevant; ultimately interested in phase changes

Total electron content:

\[
\text{TEC} = \int_{\text{GPS}}^{\text{LEO}} N_e dl
\]
The *ionospheric linear combination* of phase paths $L_1$ and $L_2$:

$$L_1 - L_2 \approx \int_{LEO}^{GPS} (1 - \frac{C}{f_1^2 N_e}) dl_1 - \int_{GPS}^{LEO} (1 - \frac{C}{f_2^2 N_e}) dl_2$$

- Clock errors and relativity terms cancel

$$\text{TEC} \approx \frac{f_1^2 f_2^2 (L_1 - L_2)}{C(f_1^2 - f_2^2)} (+ \text{arbitrary constant})$$

Assumptions:

- Same integration paths ($dl_1 = dl_2 = dl$) — straight lines
- Ignoring higher order ionospheric terms
The COSMIC Data Analysis and Archival Center (CDAAC) collects about 3000 TEC arcs per day from six COSMIC satellites.

Useful for data assimilation into space weather models — another story.
Principle equations — Abel transform

\[ T = \Delta \text{TEC} = \text{solid} - \text{dashed} \]

\[ T(p) = 2 \int_p^{p_{\text{LEO}}} \frac{r N_e(r)}{\sqrt{r^2 - p^2}} \, dr \quad \iff \quad N_e(r) = -\frac{1}{\pi} \int_r^{r_{\text{LEO}}} \frac{dT/dp}{\sqrt{p^2 - r^2}} \, dp \]

Assumptions:

- Spherical symmetry (no horizontal electron density gradients)
- Straight-line signal propagation
- Circular satellite orbits
Principle equations — Abel transform

\[ T = \Delta TEC = \text{solid} - \text{dashed} \]

\[ T(p) = 2 \int_{p}^{p_{\text{LEO}}} \frac{r N_e(r)}{\sqrt{r^2 - p^2}} \, dr \]

\[ N_e(r) = -\frac{1}{\pi} \int_{r}^{r_{\text{LEO}}} \frac{dT/dp}{\sqrt{p^2 - r^2}} \, dp \]

Upper boundary problem:

- \( dT/dp \to -\infty \) for \( p \to p_{\text{LEO}} \equiv r_{\text{LEO}} \)
- \( p^2 - r^2 \to 0 \) for \( p \to r \)
- How do we calculate \( N_e(r) \) when \( r \approx r_{\text{LEO}} \)?
Alternative method — onion peeling

\[ T = \Delta \text{TEC} = \text{solid} - \text{dashed} \]

\[ T(p_i) = \sum_{k=1}^{m} 2 \int_{p_{i+k-1}}^{p_{i+k}} \frac{r N_e(r)}{\sqrt{r^2 - p_i^2}} dr \Rightarrow N_e(p_i) = c_{i,0}^{-1} \left( \frac{T(p_i)}{p_i} - \sum_{k=1}^{m} c_{i,k} N_e(p_{i+k}) \right) \]

- Assume \( N_e(r) \) varies linearly with \( r \) in between discrete levels

- \( N_e(p_i) \) is solved iteratively starting from the top (orbit radius)

- How do we calculate \( N_e(p_{\text{LEO}}) \)?
Auxiliary measurements available (dashed lines)

\[ \Delta \text{TEC} = \text{solid} - \text{dashed} \]

\[ \Delta \text{TEC}(p) \approx 2 \sqrt{2 p_{\text{LEO}} N_e(p_{\text{LEO}})} \sqrt{p_{\text{LEO}} - p} \quad \text{for} \quad p \approx p_{\text{LEO}} \]

Fit a straight line to \( \Delta \text{TEC} \) as a function of \( \sqrt{p_{\text{LEO}} - p} \) for the uppermost few km — slope of straight line gives \( N_e(p_{\text{LEO}}) \)
• Auxiliary measurements not available

\[ \Delta TEC = \text{solid} - \text{dashed} \quad \text{(normalized to zero at the top)} \]

• Assume exponential decay (with scale height \( H \)) of \( N_e(r) \) above \( r_{\text{LEO}} \)

\[
\Delta TEC(p) \approx \sqrt{2p_{\text{LEO}}N_e(p_{\text{LEO}})} \sqrt{p_{\text{LEO}} - p} + \sqrt{\frac{\pi p_{\text{LEO}}}{2H}} N_e(p_{\text{LEO}})(p_{\text{LEO}} - p)
\]

• Fit a functional expression to obtain \( N_e(p_{\text{LEO}}) \) and \( H \)
• Auxiliary measurements not available

\[ \Delta \text{TEC} = \text{solid} - \text{dashed} \quad \text{(normalized to zero at the top)} \]

Compensate for the missing auxiliary-side TEC:

\[ T(p) \approx \Delta \text{TEC} + N_e(p_{\text{LEO}}) \sqrt{\frac{\pi}{2} p_{\text{LEO}} H} \left[ 1 - \exp \left( \frac{p_{\text{LEO}} - p}{H} \right) \text{erfc} \left( \sqrt{\frac{p_{\text{LEO}} - p}{H}} \right) \right] \]
Summary of retrieval approach

COSMIC

- Subtract aux.−side TEC
- Estimate $N_e(r_{LEO})$

CHAMP

- Subtract top TEC
- Estimate $N_e(r_{LEO})$ & $H$
- Compensate for missing aux.−side TEC

Onion peeling $\rightarrow N_e(r)$
Five COSMIC satellites flying in almost identical orbit planes while receiving signals from the same GPS satellite (PRN 14)

Five profiles measured 8-10 minutes apart within 1000 km of each other

Are we measuring ionospheric dynamics (e.g., TID)?
Some possible applications

- Verification of other ionospheric data
- Verification of ionospheric models
- Studies of profile variability
- Ionospheric storm studies
- Studies of seasonal variations

Caveats:
- GPS phase cycle-slips
- Spherical symmetry assumption
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<thead>
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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$e$</td>
<td>elementary charge</td>
<td>$\approx 1.602 \times 10^{-19}$ C</td>
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<td>$m$</td>
<td>electron mass</td>
<td>$\approx 9.109 \times 10^{-31}$ kg</td>
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<tr>
<td>$\varepsilon_0$</td>
<td>permittivity of vacuum</td>
<td>$\approx 8.854 \times 10^{-12}$ Fm$^{-1}$</td>
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<tr>
<td>$p$</td>
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Further reading


