Horizontal refractivity gradients in the atmosphere introduce errors in retrieved profiles from radio occultation measurements. These errors are introduced because the profiles are retrieved using the assumption of spherical symmetry and interpreted as vertical profiles. However, a more correct interpretation would be to think of them as a mapping of the two-dimensional structure of the refractivity into a one-dimensional profile. For data assimilation purposes this is an important realization, because error covariances may be significantly reduced if the retrieved profiles are interpreted in a way more closely related to the manner in which the data are obtained and processed. In this paper we assess close to worst-case errors in retrieved refractivity profiles by simulating the occultation measurements in cases where the signals propagate through a model of a weather front, including moisture. The synthetic occultation data are inverted using the assumption of spherical symmetry to obtain retrieved refractivity profiles. We compare the retrieved refractivity profiles with profiles obtained from the frontal model in three different ways: i) refractivity values at the tangent points, ii) refractivity values averaged over 2° of longitude in the occultation plane, and iii) a mapping of the two-dimensional refractivity field in the occultation plane into a one-dimensional profile. We show that the latter method reduces the errors by about 80%.

Key words: radio occultation, refractivity, horizontal gradients, data assimilation.

1. INTRODUCTION

Retrieved profiles from radio occultation measurements are based on the assumption of spherical symmetry in the atmosphere. Thus, horizontal gradients in the atmosphere will introduce errors if the retrieved profiles are interpreted as vertical profiles near the ray path tangent points. Many authors have addressed this problem and assessed the influence of horizontal gradients on retrieved profiles of refractivity, pressure, temperature, and geopotential height from radio occultation refractometry measurements (Gurvich & Sokolovskiy 1985; Gorbunov 1988, 1990, 1996; Hardy et al. 1994; Kursinski et al. 1997; Ahmad & Tyler 1998, 1999; Zou et al. 1999, 2002; Syndergaard 2000; Healy 2001; Gorbunov & Kornblueh 2001). The most comprehensive error analysis (Kursinski et al. 1997) showed that the error in the retrieved refractivity profiles, due to along-track gradients (horizontal gradients in the occultation plane), are dominating below 25 km altitude. Statistically, between 10 and 25 km, the fractional error standard deviation is about 0.2%, whereas below 10 km it increases to become about 1% near the surface, mostly as a result of horizontally inhomogeneous moisture distributions in the lower troposphere.

In this paper we investigate worst-case refractivity errors in the presence of a weather front. Using simulations, we assess the errors if the retrieved profiles are compared to profiles following the loci of the ray path tangent points. Due to a very simplified geometry in our simulations, the loci of the tangent points are aligned vertically. However, in a more general radio occultation geometry, the errors (differences between tangent point profiles and vertical profiles) introduced because the tangent points are drifting during the occultations are significant as well. By interpreting the retrieved profiles as profiles along the loci of the tangent points, errors are generally smaller than if the retrieved profiles are interpreted as vertical profiles at a single latitude and longitude position (Foelsche & Kirchengast 2002). An even more correct interpretation is to look at a retrieved profile as a mapping of the two-dimensional (2D) refractivity field in the occultation plane into a one-dimensional (1D) profile. Using a simple linear mapping operator, we show that such an interpretation reduces errors (differences between mapped and retrieved profiles) further. We also compare the retrieved profiles to simple horizontal averages and show that such an approach does not reduce the errors significantly. The 2D to 1D mapping can be used as an observation operator for future data assimilation of refractivity profiles from radio occultations into numerical weather prediction models. As such it seems very promising, being both accurate, simple, and fast.
Figure 1. Cross-section of the refractivity field of the model weather front used in the simulations (contours indicated by solid curves). Every 25th ray path from an occultation simulation is superimposed (dashed curves). The diamonds indicate the actual tangent points of the ray paths.

Figure 2. Illustration of the geometry used in the simulations. The orbits of the transmitter (T) and the receiver (R) are identical, but the satellites are moving in opposite directions. The assumed tangent points are aligned vertically as indicated by the line in the middle. The dashed curves indicate ray paths.

2. ERROR ASSESSMENT APPROACH

To assess the worst-case errors in retrieved refractivity profiles due to horizontal gradients we have developed a parametric model of a weather front, including water vapor. A cross-section of the refractivity field around the front is shown in Figure 1. The surface frontal boundary is situated at 0° longitude. In the following we describe four different ways to obtain a refractivity profile from this field. Profiles obtained as described in Section 2.1 are then compared to profiles obtained as described in Sections 2.2–2.4. In Section 3 the differences are denoted as errors in the retrieved profiles. Alternatively, and conceptually more correct for data assimilation purposes, the differences (with opposite signs) can be looked upon as errors in the other three approximate ways of modeling a retrieved profile.

2.1. The Retrieved Profile

Using the Radio Occultation Simulation for Atmospheric Profiling (ROSAP) high-accuracy, three-dimensional ray tracing code (Høeg et al. 1995), we first obtain simulated phase measurements. We choose a very simple occultation geometry, with the transmitter and the receiver in co-planar circular orbits (coincident with the Earth’s equatorial plane) at equal altitudes and moving away from each other (Figure 2). Signals are propagated from the transmitter (T) to the receiver (R) while the satellites move such that the ray paths penetrate deeper and deeper into the atmosphere. The orbits are chosen such that the “assumed” ray path tangent points are located at 0° longitude. The actual tangent points may be shifted slightly with respect to the assumed tangent points (cf. Section 2.2). The signals are refracted toward the Earth due to the vertical refractivity gradients. For the ray paths with tangent points in the lowest part of the troposphere the total bending angles are around 1°. Ray paths from an occultation simulation are superimposed on the model weather front in Figure 1, where also the actual tangent points are indicated (note that ray paths become parabola-like curves in a longitude-altitude coordinate system).

Assuming spherical symmetry, the obtained phase measurements are then processed to bending angles and impact parameters (e.g., Melbourne et al. 1994). The bending angles are inverted to retrieve the refractivity profile via the Abel transform (e.g., Fjeldbo et al. 1971). This process is also based on the assumption of spherical symmetry. The parameters of the frontal model were chosen to avoid multi-path in the simulations due to the use of ray tracing in constructing the simulated phase measurements.

2.2. The Tangent Point Profile

We refer to the assumed tangent point profile as the values in a vertical line at 0° longitude. In general, the assumed tangent points are the ones that can be derived geometrically from the transmitter and receiver positions and the corresponding retrieved impact parameters and ray tangent altitudes, assuming spherical symmetry. In our simulations they are aligned vertically only because of the simplified occultation geometry. The actual tangent points are in general shifted slightly with respect to the assumed tangent points due to the influence of the horizontal gradients on the ray paths (cf. Figure 1). Since the
actual tangent points cannot be estimated from the data alone, we consequently use the assumed tangent points as the reference in the comparisons in Section 3.

2.3. The 2° Average Profile

A 2° horizontal average is obtained simply by averaging the model field over the range \(-1°\) to \(+1°\) longitude, at fixed altitudes.

2.4. The 2D Mapped Profile

A weighted average of the refractivity field is computed as briefly outlined below. In essence, the weighted average is a linear mapping of the 2D refractivity structure in the occultation plane into a 1D profile, mimicking the observation geometry as well as the subsequent data inversion, assuming spherical symmetry. More precisely, it consists of the integration of the refractivity along finite straight lines parallel to the propagation directions centered at the assumed tangent points, followed by a finite straight-line Abel transform equivalent inversion. Assuming finite straight lines would be a crude approximation if we were to simulate the observations by such an approach. However, it is a feasible approximation when it is made in both the forward and the inverse part of the mapping, since the main part of the errors due to this approximation cancels out.

Let the model refractivity field be denoted by \(N\), and a finite line integral through this field, centered at the assumed tangent point at a fixed altitude, be denoted by \(\beta\). A profile (vector) of line integrals, \(\beta\), is then given by

\[
\beta = B(N) ,
\]

where \(B\) is the forward part of the mapping function. Having \(\beta\), the mapped profile, \(\bar{N}\), is obtained as

\[
\bar{N} = A(\beta) ,
\]

where \(A\) is the inverse part of the mapping function. Because of the straight-line assumption, both \(A\) and \(B\) are linear. A forthcoming paper describes the mapping functions \(A\) and \(B\) in more detail and they are discretized to obtain the elements of matrices \(A\) and \(B\). As an illustration of the combined mapping function, the weights (essentially the elements of the matrix \(AB\)) for calculating the mapped value at 5 km, are shown in Figure 3. The pattern of the largest weights follows straight lines in a spherical atmosphere. These lines are limited to a maximum length of about 715 km, and bounded by a 30 km upper altitude. In practice we use a fast recursion formula to calculate the mapped profiles and a higher resolution than that indicated by Figure 3.

Figure 3. Weights for the mapping on a regular grid in longitude-altitude coordinates, and for a tangent point value of 5 km. Red dots indicate positive weights and green dots indicate negative weights. The size of the dots indicate the absolute value on a logarithmic scale.

3. PROFILE COMPARISONS

Figure 4 shows the refractivity profile at the tangent points together with the horizontal average, the profile obtained with the 2D mapping approach, and the profile from the retrieval of the simulated phase data. For the horizontal average, a 2° horizontal bin was used since it corresponds to the straight-line path through a 1 km vertical thickness of the atmosphere near the tangent point. The fractional differences between the retrieved profile and the three other profiles are shown in the lower panel of Figure 4. It is clear that the error is much smaller when the 2D mapped profile is used for the comparison. We assessed the errors in several other cases, including cases where the orientations of the frontal model were such that horizontal gradients perpendicular to the propagation direction had influence too. In all cases we see a reduction of the errors of about 80% or more when comparing the retrieval to the mapped profile as opposed to the tangent point profile. We also calculated 6° simple horizontal averages in the same manner as the 2° horizontal averages, corresponding approximately to the lengths of the finite lines used in the 2D mapping. However, the results were generally further from the retrieval than the tangent point profiles and the 2° averages.

It should be emphasized that the errors shown in Figure 4 are close to worst-case errors in the absence of multi-path propagation. Several of the lowest ray paths shown in Figure 1 travel along or very near the elevated frontal surface on the left hand side, but travel through a relatively horizontally homogeneous atmosphere on the right hand side.

Figure 5 shows a case with the model atmosphere
shifted by 1° of longitude toward the right, relative to that shown in Figure 1. In this case there are notable horizontal gradients at the tangent points as high as

Figure 5. Similar to Figure 1, but with the refractivity field shifted by 1° of longitude.

5 km, as opposed to the case in Figure 1 where horizontal gradients are appreciable only below about 3 km. The upper panel of Figure 6 shows the refractivity profile for the various representations. As in Figure 4 the tangent point profile has the greatest amount of vertical structure whereas the retrieved profile is considerably smoother. Although the 2° average has less vertical variation than the tangent point profile, the uniform weighting of the model atmosphere between −1° and +1° is still inappropriate when compared to the retrieved profile.

4. DISCUSSION AND CONCLUSION

We have demonstrated close to worst-case errors due to atmospheric horizontal gradients in retrieved profiles of refractivity from radio occultation measurements. These errors are on the order of 5 % if the retrieved profiles are interpreted as profiles following the tangent points. However, if the retrieved profiles are interpreted as a 2D weighted average, the errors are reduced by a factor of about 5. In essence, the weighted average is a linear mapping of the 2D refractivity structure in the occultation plane into a 1D profile, mimicking the observation geometry as well as the subsequent retrieval.
The simulations in this paper indicate that the error standard deviation of refractivity profiles, and probably any systematic bias as well, due to the horizontal gradients, can be significantly reduced simply by interpreting retrieved profiles differently. This would be a very significant improvement of the error characteristics for future data assimilation purposes. In fact, when assimilating refractivity profiles, the retrieved 1D profiles should, conceptually, be interpreted as being the true mapping in the occultation geometry of the three-dimensional atmosphere, the only errors in the profiles then being due to measurement errors (not considering ionospheric residual errors, upper boundary errors, satellite velocity errors, etc.). With such an interpretation, differences between mapped (e.g., using a linear approximate mapping as in this work) and retrieved profiles, due to horizontal variations, should be attributed to the mapping operator as representativeness errors, i.e. the limitation of the mapping operator to represent the retrieved profiles in the absence of measurement errors.

A recent study compared retrieved profiles from the GPS/MET experiment to profiles obtained from the European Centre for Medium-range Weather Forecasts (ECMWF) analyses fields, by a full cycle of simulations, i.e., including the simulation of phase and amplitude data by a wave optics propagator, and a subsequent inversion process identical to the one performed on the GPS/MET data (Gorbunov & Kornblueh 2001). This is in principle a more correct way to compare the profiles. However, it is of limited practical use for data assimilation purposes, because a full simulation cycle, in particular the wave propagation, is computationally expensive. The approach by Gorbunov & Kornblueh also involves the inclusion of a climatological model above the levels of the ECMWF analyses, which complicates the implementation.

Our mapping operator is somewhat similar to the mapping function described as a 2D resolution kernel by Ahmad & Tyler (1998). The 2D resolution kernel was later suggested as a means for reducing the errors of refractivity profiles from GPS radio occultation measurements (Bealy 2001). However, a fundamental difference is that our mapping approach, without compromising the accuracy significantly, is based on the integrations along finite straight lines as opposed to an infinite double integral describing Ahmad & Tylers 2D resolution kernel. Furthermore, the mapping operator can easily be adapted to a more general occultation geometry where the tangent point locations and propagation directions may vary as a function of the altitude.

In general, our mapping approach has several features that makes it very attractive as an observation operator for future data assimilation of radio occultation data:

- It is simple to implement.
- The computations are fast.
- The mapping of refractivity can be described as a linear matrix operator.
- There is no need for the inclusion of climatology above the levels of the model.

In the hypothetical case of a spherically symmetrical atmosphere, the mapping is designed to be exact, i.e. the 2D to 1D mapping will result in the profile of the spherically symmetrical refractivity field. When horizontal gradients are present, the residual error of the mapping is mainly due to two separate reasons: 1) the finite straight-line assumption, and 2) centering the line integrals at the assumed tangent points instead of the actual tangent points. To assess the separate contributions from these two approximations we additionally calculated mapped refractivity profiles in cases where the line integrals were centered at the actual tangent points. The actual tangent points were obtained from the ray-tracing simulations, and it should be emphasized that in an operational data assimilation scheme it would be very computationally expensive to estimate them. In Figure 7a the curve labeled “Assumed” is identical to the curve labeled “2D mapped” in Figure 4, whereas the curve labeled “Actual” has been obtained in a similar way, though centering the line integrals at the actual tangent points in the mapping approach. In our simulations, the maximum shift of the actual tangent points relative to the assumed ones were about 10 km (cf. Figure 1). Figure 7b shows similar curves for a case where the model weather front was turned 45° such that horizontal gradients perpendicular to the propagation direction also had some influence on the ray paths. As seen, there is no significant improvement when using the actual tangent points as opposed to the assumed tangent points. Although the maximum errors are reduced by a factor of ~2 in the cases shown, we cannot conclude that it would be of any advantage in general to center the line integrals at the actual tangent points – if they could be estimated. It can be concluded, however, that the remaining residual (dotted curves) must be due to the finite straight-line approximation, since there are no other significant approximations involved. These plots therefore indicate that the finite straight-line approximation and the use of approximate (assumed) tangent points result in residual errors of about the same size. It should be noted that the tangent points may in general have a horizontal drift on the order of 100 km, mainly due to the specific occultation geometry and the relative movement of the satellites (Hoeg et al. 1995). The plots therefore also indicate that comparisons of retrieved profiles with mapped profiles, not taking into account the geometrical horizontal drift of the tangent points, are not generally sufficient; it could yield errors 10 times larger than the differences between the dashed and dotted curves in Figure 7.

Although we intentionally avoided multi-path in the simulations, we expect the mapping to work equally well in most cases where there would be multi-path propagation. As long as the observed bending angles are corrected for diffraction and multi-path, e.g.,
Figure 7. Refractivity errors as a function of the altitude when the mapping is centered at the assumed tangent points (dashed) and when the mapping is centered at the actual tangent points (dotted). a) The case for the front as in Figure 1. b) A case where the orientation of the front is different by 45° relative to that shown in Figure 1, but otherwise the same.

using the back-propagation approach (Gorbunov & Gurvich 1998), in combination with the canonical transform method (Gorbunov 2002), the retrieved refractivity profiles should in principle be comparable to the mapped profiles. However, severe horizontal gradients can cause the reconstruction of the bending angle versus impact parameter to be multi-valued, even after multi-path correction using the canonical transform (Gorbunov 2002). The Abel transform used to obtain the retrieved refractivity profile can not be correctly applied in those cases. Similarly, the linear mapping operator is based on the assumption of a monotonic sequence of the tangent point altitude versus horizontal location.

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