Modeling and inverting radio occultation signals in the moist troposphere

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Abstract. Accurate modeling of radio occultation signals is performed by solving the Helmholtz equation with the use of a multiple-phase-screen technique. Refractivity is assumed spherically symmetric, and vertical profiles are reproduced from high-resolution tropical radiosondes. As a result, the characteristics of the signals, which are important for their tracking in low Earth orbit, are evaluated: the spectral bandwidth, \( \sim 50 \) Hz, and the random phase acceleration, \( \sim 1000 \) Hz/s. The complex signals are inverted with the use of two radio holographic methods: back propagation and sliding spectral (radio optics). For the back propagation method, finding the position of the auxiliary trajectory which provides an unambiguous bending angle function of impact parameter appears to be a problem. For the sliding spectral method a simple technique, which takes into account the whole spectral content of the signal without identification and selection of local spectral maxima, is introduced and tested. The sliding spectral method allows for the stable reconstruction of bending angles and refractivity with vertical resolution of \( \sim 0.5 \) km. The small-scale laminated structure of refractivity results in propagation of radio occultation signals down to significantly lower observation altitudes than in the case of smooth refractivity. Information content of radio occultation signals at those low altitudes is important for the radio holographic inversions.

1. Introduction

The radio occultation (RO) technique, which had been originally developed for planetary atmospheres [Eshleman, 1973], has been proposed and then successfully applied for the Earth’s atmosphere (Gurvich and Krasil’nikova [1987], Yunck et al. [1988], Hardy et al. [1992, 1994], Melbourne et al. [1994], Kursinski et al. [1997], Rocken et al. [1997], Hoče [1997], Steiner et al. [1999], Feng and Herman [1999], etc.). A history of the GPS RO sounding is given by Yunck et al. [2000]. The first observational data were obtained and processed in the Global Position System / Meteorology (GPS/MET) experiment [Ware et al., 1996; Kursinski et al., 1996]. Comparisons of the GPS/MET-retrieved refractivities to radiosonde and numerical weather prediction (NWP) model data showed statistically good agreement at all altitudes except in the lower troposphere [Rocken et al., 1997]. In the lower troposphere, GPS/MET data are often significantly corrupted, and retrieved refractivities are statistically negatively biased with respect to both radiosonde and NWP model data. This corruption and bias depend on latitude; they are much larger in the tropics than in polar regions. Significant effort has been invested to eliminate or to reduce this bias by applying diffractional back propagation methods to disentangle multiple rays in the lower troposphere [Karayel and Hinson, 1997; Gorbuñov and Gurvich, 1998; Mortensen and Hoeg, 1998]. However, the application of those methods did not reduce the observed negative bias in retrieved refractivity.

The input data for inversion algorithms are raw phases (in excess of the phase delay in a vacuum) and amplitudes of the L1 and L2 GPS carrier frequencies, provided as functions of positions and velocities of

\[ \text{Reference} \]
Figure 1. Examples of GPS/MET radio occultation data. (left) Polar occultation. (right) Tropical occultation.

The satellites. Figure 1 shows two typical examples of the GPS/MET 50 Hz data, L1 excess Doppler and signal-to-noise ratio (SNR), which is proportional to amplitude, for polar (left graphs) and tropical (right graphs) regions. As can be seen, the structure of the signal in the tropical troposphere is much more complicated than at higher altitudes and latitudes. This is due to complicated vertical structure of the refractivity caused mainly by water vapor. A sharp increase of the magnitude of Doppler and amplitude fluctuation, i.e., the spread of the spectrum of the RO signal, as it descends into the moist troposphere (this is visible in the right graphs in Figure 1 after ~45 s), is typical for tropical occultations.

The RO signals shown in Figure 1 were obtained on output of the GPS/MET receiver that was tracking signals by means of digital phase-locked loop (PLL) [Thomas, 1995]. Since its performance has not been tested by signals with structure adequate for propagation through the moist troposphere, it is not yet clear how the phase and amplitude of radio waves arriving at the antenna are related to the phase and amplitude of the signal on output of the receiver and whether complicated structure of the input signal could cause corruption of the output signal. An evidence of such corruption follows from the difference in penetration depth of occultations and in magnitude of the refractivity bias for two observational prime times [Kuo et al., 2000a]. Between those prime times the GPS/MET receiver tracking firmware had been updated by Jet Propulsion Laboratory (JPL). Since the lower tropospheric data contain information about water vapor, which is very important for NWP models, it is necessary to minimize the probability of getting corrupted data from future missions and to test the receivers by means of simulated signals which take into account the complicated vertical structure of refractivity in the moist troposphere. It is also important to test the inversion algorithms with the use of those signals.

In this paper we use high-resolution tropical radiosonde data (which capture the complicated ver-
tical structure of humidity in the lower troposphere with a vertical resolution of ~20-30 m) for simulations of the diffracted electromagnetic field at L1 GPS carrier frequency, which would be received at an altitude of ~750 km (3000 km beyond the limb). The refractivity is treated as spherically symmetric, the atmosphere is represented by a number of parallel phase screens, and the Helmholtz equation is solved in free space between the screens by expansion of the solution in a series of plane waves [Knepp, 1983; Martin and Flatte, 1988; Martin, 1993; Karayel and Hinson, 1997; Gorbunov and Gurvich, 1998]. The structure of the simulated signals is evaluated through their spectra and phase acceleration. Two radio holographic methods previously discussed in the literature, back propagation and radio optics (sliding spectral), are tested for inversions of the modeled complex RO signals, i.e., for reconstruction of the bending angles (which then are used for reconstruction of the refractivities). The back propagation method is used without detailed discussion of the technique, by referring to available publications. For the sliding spectral method this paper introduces a simple technique that takes into account the whole spectral content of a RO signal without an identification and selection of local spectral maxima, and this technique is discussed in more detail.

2. High-Resolution Radiosonde Data

For modeling RO signals we use high-resolution radiosondes [Steurer, 1996] in the tropics. Those radiosondes use a 4 Hz internal sampling rate (for both thermistor, hygriostor, and pressure sensor). Every 24 samples are averaged and downlinked at 1/6 Hz sampling rate. Output data contain geopotential altitude $h_p$ (km), pressure $P$ (mbar), temperature $t$ (°C), and relative humidity $\nu$ (%) with 6 s time increment. For this study only small-scale structure of refractivity is important; thus the difference between geopotential altitude $h_p$ and geometric altitude $h$ does not matter. The left graph in Figure 2 shows the vertical profiles, $\nu(h)$ and $t(h)$, obtained from the radiosonde launched from Majuro Island in the Pacific Ocean (7.1°N, 171.4°E) on October 1, 1995, 1200 UTC. As seen, the vertical structure of humidity is very complicated. The right graph shows $\nu$ and $t$ samples in the altitude interval 1-3 km around the dip in humid-

![Figure 2](image-url)

Figure 2. Relative humidity $\nu$ and temperature $t$ profiles from high-resolution tropical radiosonde. (left) Here, $\nu$, solid curve; $t$, dashed curve. (right) Radiosonde samples around the dip in humidity at 2 km (indicated by arrow in left graph): $\nu$, squares; $t$, circles.
ity indicated by the arrow in the left graph. As seen, this dip is well resolved by the radiosonde samples. Also, it is seen from the right graph that the breakpoints in $\nu(h)$ are clearly correlated with the breakpoints in $t(h)$, as it should be when the radiosonde is ascending through a cloud system. The hygristor error must be within 5% in $\nu$ when $\nu > 30\%$ [Pratt, 1985]. However, as can be seen from $\nu(h)$ between 1.2 and 1.7 km (right graph in Figure 2), the random noise level in $\nu(h)$ is smaller than 5%, while bias error in $\nu(h)$ (as well as in $t(h)$) does not matter for this study. Another important parameter of the hygristor is the response time which is $\sim$1-3 s at $t = 0^\circC$ and $\sim$30 s at $t = -30^\circC$ [Pratt, 1985]. Thus, below $\sim$5-6 km ($t \gtrsim -5^\circC$) the response time is not larger than the 6 s sampling interval. For a mean radiosonde ascent rate this corresponds to $\sim$20-30 m vertical resolution (as seen from the right graph of Figure 2). All this provides enough evidence that the vertical structure of humidity with scales of $\sim$0.1 km reproduced by the high-resolution radiosondes below $\sim$6 km is realistic.

At first we calculate pressure of water vapor $P_w$ [Garratt, 1992]

$$P_w = \nu E = \nu \times 6.11 \exp\left(17.67 \frac{T - 273.15}{T - 29.65}\right),$$

where $E$ (mbar) is the saturated pressure of water vapor, given temperature $T$ (K). Then we calculate refractivity $N$ [Bean and Datton, 1968]

$$N = 77.6P/T + 3.73 \times 10^5 P_w/T^2$$

as a function of the geometric altitude $h$.

Figure 3 shows three vertical refractivity profiles $N(h)$ obtained from the Majuro Island radiosondes on 3 days, October 1, 9, and 19, 1995, 1200 UTC. The refractivity profiles are smooth above $\sim$8 km. Below $\sim$6 km the structure of refractivity becomes very complicated. It includes the subrefraction ($dN/dh > 0$) and the superrefraction ($dN/dh < -10^6/R_E \approx -160$ N units/km, where $R_E$ is the Earth’s radius). The arrow in Figure 3 indicates the dip in refractivity caused by the dip in humidity shown in the right graph in Figure 2.

Figure 3 also shows two analytic refractivity models, models A and B, that are used for comparisons and for validation of the forward propagation and inverse techniques,

$$N_A(h) = 400 \exp\left(-h/8\text{ km}\right),$$

$$N_B(h) = N_A(h) \left[1 - 0.05 \left(\frac{2}{\pi}\right) \tan^{-1}\left(\frac{h - 3\text{ km}}{0.05\text{ km}}\right)\right].$$

Figure 3. Refractivity profiles used for simulations. Profiles A and B, models; profiles 1, 2, and 3, high-resolution radiosondes. Profiles 2, 3, A, and B are shifted by 50, 100, 150, and 150 N units for display purposes.
For model B $dN/dh \approx -209$ N units/km at $h = 3$ km, which corresponds to the superrefraction conditions.

3. Phase Screen Model

To solve the wave propagation problem, we assume a transmitter infinitely far in space. This is equivalent to a plane wave entering the atmosphere, as shown in Figure 4 (the large distance to the GPS satellite, as compared to the vertical scale of the atmosphere, allows this approximation). We replace the three-dimensional (3-D) atmospheric refractivity by a number of parallel phase screens normal to the direction of the incident wave, as shown in Figure 4, and we “observe” the signal on the plane at a distance $l_0 = 3000$ km from the limb. Approximation of the phase screen is applicable when the deviation of rays from the straight lines inside a refractive medium is smaller than the smallest scale of the refractivity irregularities that are significant for a given propagation problem. Thus the approximation of the whole atmosphere by a single phase screen is not applicable. Instead, multiple phase screens have to be used, so that the aforementioned condition is satisfied between the adjacent screens.

The excess phase (phase path) $s$, assigned to a point on a phase screen, is equal to refractivity integrated between the adjacent screens along the straight line normal to the screens. We assume that refractivity $N(h)$ is spherically symmetric with the vertical profile given by radiosonde (and exponential extrapolation above). Since the Fresnel zone on a phase screen is much smaller than the Earth’s radius, instead of the 3-D problem we consider a 2-D problem in Cartesian coordinates $(y, z)$, as shown in Figure 4. For a small enough distance between the adjacent screens $\Delta l$ (which will be the case) it is sufficient to calculate the excess profile $s(z)$ approximately, by multiplying $\Delta l$ by the refractivity profile $N(h)$, taking into account the shift $z_{sh}$ and tilt $\theta$ with respect to the phase screen at a distance $l$ from the limb, as shown in Figure 4,

$$s(z, l) = 10^{-6} N \left( \frac{z - z_{sh}}{\cos \theta} \right) \Delta l,$$

where $z_{sh} = R_E (\cos \theta - 1)$ and $\cos \theta = (1 - l^2/R_E^2)^{1/2}$.

After the excess phase on the screen is calculated with the discreteness provided by the radiosonde data, it is interpolated onto a much denser grid with the increment $\Delta z = 1$ m (by means of log cubic spline interpolation). This is necessary to allow sufficient bandwidth of the angular spectrum of diffracted radio waves (discussed in section 4).

Treatment of the refractivity structures reproduced by high-resolution radiosondes as spherically symmetric is, of course, an approximation. The spatial structure of refractivity in the moist troposphere is very complicated, and its statistical characterization (e.g., through structural functions) is not well known yet. The coefficient of anisotropy of the refractivity irregularities (i.e., ratio of their horizontal-to-vertical scales) may depend on the scale of the irregularities, altitude, region, and weather conditions. Here we use the spherically symmetric assumption.

Figure 4. Layout of the wave propagation problem in the atmosphere with the use of the multiple phase screens.
in the forward propagation tropospheric problem as the first step of this study. This assumption may be substantiated by the following: There is evidence of laminated structure of moisture in the tropical troposphere, obtained from airborne lidar soundings, at altitudes 2-8 km, with horizontal scales of ~100 km, and vertical scales of ~1 km, or less (D. Lenschow, personal communication, 2000). Our simulations show that given a vertical structure of the refractivity irregularities, the larger their horizontal correlation distance is, the larger are the fluctuations of the phase and amplitude of the RO signal. Thus the spherically symmetric assumption is a “worst case” in terms of the signal structure. Modeling of radio occultation signals is important for the development of tracking techniques which would process those signals with minimal corruption. Thus testing receivers for the future radio occultation missions with the “worst case” signals is useful to gain confidence in their performance under real conditions. The spherically symmetric assumption is also useful for testing the inversion techniques which use the same assumption. It allows one to separately study the retrieval errors which are fundamentally related to the multi-path propagation and to limited vertical resolution of the inversion techniques, without mixing them with the errors resulting from breaking the assumption of spherical symmetry, i.e., from horizontal inhomogeneity of refractivity. Naturally, for more accurate error characterization of the RO technique in the troposphere, more realistic simulations with 2-D (3-D) refractivity models are necessary.

4. Modeling of RO Signals by Forward Propagation

To solve the wave propagation problem, i.e., the Helmholtz equation, we apply the multiple-phase-screen technique, which is equivalent to the split-step technique for the parabolic equation. This technique has been widely used, mainly for wave propagation in random media [Knepp, 1983; Martin and Flatte, 1988; Martin 1993]. It has been also applied for modeling of direct RO problem in the Earth’s atmosphere [Karayel and Hinson, 1997; Gorbunov and Gurvich, 1998]. The complex amplitude of the electromagnetic field on input to a screen $u_{in}(z)$ is related to the amplitude on output of the screen $u_{out}(z)$ by

$$u_{out}(z) = u_{in}(z)a_w(z)\exp[iks(z)], \quad (5)$$

where $k = 2\pi/\lambda$ is a wave number, $\lambda$ is a wavelength, $s(z)$ is the excess phase assigned to the screen, and $a_w(z)$ is a real windowing function, which will be discussed below. We use $\lambda = 19.029\ldots$ cm, which corresponds to L1 GPS signal. Expanding $u_{out}(z)$ into Fourier series yields

$$u_{out}(z) = \sum_{m=-M/2}^{+M/2} c_m \exp(2\pi imz/H), \quad (6)$$

where $H$ is the vertical extent of the phase screen and $M$ is the number of data on the screen. Each harmonic in (6) is associated with the plane wave in space, having the vertical component of a wave vector $k_y = 2\pi m/H$ and thus the horizontal component $k_y = |k^2 - (2\pi m/H)^2|^{1/2}$. Then the complex amplitude of the electromagnetic field in free space beyond the screen may be represented as the sum of the plane waves

$$u(y, z) = \sum_{m=-M/2}^{+M/2} c_m \exp\{2\pi imz/H + i(y - y_s[k^2 - (2\pi m/H)^2]^{1/2})\}, \quad (7)$$

where $y_s$ denotes the position of the phase screen. The complex amplitude $u(y, z)$ is calculated either on the next screen $y_{s+1} = y_s + \Delta l$ (then it is used as the input amplitude $u_{in}(z)$, and the whole procedure given by (5)-(7) is repeated recursively) or on the observational plane (trajectory) $y = l_0$ after propagation through the last phase screen. On input to the first phase screen $u_{in}(z) = 1$. To speed up the computations, the fast Fourier transform (FFT) [Press, 1995] is applied to calculate $c_m$. It is important that all phase screens and the observational trajectory are parallel. In this case, (7) may be represented by means of the same eigenfunctions as (6) and by means of the complex amplitudes

$$c'_m = c_m \exp\{i(y - y_s[k^2 - (2\pi m/H)^2]^{1/2})\}. \quad (8)$$

This allows application of the inverse FFT for summation in (7) and makes the whole procedure computationally feasible. For back propagation (discussed in section 6) the complex amplitude on the observational trajectory is expanded in the Fourier series by means of (6) and then propagated in free space back to the limb by means of (7). In this case, $y_s$ denotes the position of the observational trajectory, and $y$ denotes the position of the “auxiliary” trajectory; thus $y - y_s < 0$. 


A given vertical increment \( \Delta z \) allows one to resolve the harmonics on the phase screen with the maximal wave number \( k_z = \pi/\Delta z \). Thus \( \Delta z = 1 \) m allows one to take into account associated plane waves in space whose propagation direction differs from the initial one by \( k_z/k = \lambda/2\Delta z \approx 0.1 \) rad, which is larger than the maximum regular bending angle in the atmosphere, \( \sim 0.03-0.04 \) rad. Thus the applied vertical discreteness allows sufficient bandwidth of the angular spectrum of radio waves.

Fourier expansion of the complex amplitude on the phase screen, which has the vertical extension \( H \), implies periodic continuation of the function \( u(z) \) with the period \( H \). This may cause parasitic interference of radio waves, propagated through the adjacent periods, at a large enough distance from the limb (on the observational trajectory). To avoid this effect, the period \( H \) should be larger than the maximal-allowed angular bandwidth of the diffracted signal multiplied by the distance to the observational trajectory \( l_0 \); that is, it should be larger than \( \sim 300 \) km.

In our simulations we use \( M = 2^{19} \), i.e., \( H = 524.288 \) km, and we apply log linear extrapolation of the excess phase within the period. Another source of parasitic interference is a seam point of \( \alpha(z) \) between the adjacent periods. To suppress that effect, we apply the real windowing function \( a_w(z) \) introduced in (5).

The function \( a_w(z) \) provides smooth conversion of amplitude to zero below \( z = z_{sh} \) (\( z_{sh} \) is introduced in (4)) and above \( z = 120 \) km.

We use 2000 phase screens spaced by \( \Delta l = 1 \) km. This results in the displacement of rays from the initial propagation direction between the adjacent screens which is much smaller than 20-30 m (the smallest scale of refractivity irregularities which could be potentially resolved by the radiosondes). Thus the aforementioned condition of applicability of the phase screen model is satisfied. The adequate number of phase screens was also verified by control simulation with a larger number of screens. It appeared that the increase of the number of screens from 2000 to 4000 caused insignificant difference in the simulated signal (which resulted in \( \sim 0.1\% \) difference in bending angles after inversion of the signal by the radio holographic techniques (see section 6)).

The phase and amplitude are calculated from the complex signal: \( \phi = \arctan (\text{Im} u / \text{Re} u) \) and \( a = (\text{Re} u)^2 + (\text{Im} u)^2)^{1/2} \). The calculated \( \phi \) contains cycle ambiguity. To directly calculate the bending angles from the slope of the received phase front (under the assumption of single-path propagation), the continuous (accumulated) phase is necessary. The applied vertical increment \( \Delta z = 1 \) m allows one to resolve the cycle ambiguity, because the phase lapse is smaller than half a cycle. The accumulated phase is calculated by adding of 0 or \( \pm 2\pi \) to each reconstructed \( \phi_i \) successively, to minimize \( |\phi_i - \phi_{i-1}| \). When lower sampling rate is applied, the cycle slips are possible. However, if the sampling rate is larger than the spectral bandwidth of the signal, then, according to the sampling theorem [Proakis and Manolakis, 1992] the signal may be resampled at higher rate (by means of Fourier interpolation), and the cycle ambiguity may still be resolved.

5. Structure of RO Signals in Low Earth Orbit

We present the modeled complex RO signals as functions of altitude on the observational trajectory \( z = z_{obs} \) (see Figure 4). In our case this altitude coincides with the altitude of the tangent point of the straight line connecting transmitter and receiver, and this is convenient for comparisons to real RO observations of GPS from low Earth orbit (LEO). Doppler frequency shift \( f_d \) is proportional to receiver velocity \( v_\perp \) (see Figure 4), \( f_d = v_\perp (ds/dz)/\lambda \). We use \( v_\perp = 3.2 \) km/s, which approximately corresponds to observations from LEO at \( \sim 750 \) km altitude when GPS is in the LEO plane. The applied vertical increment \( \Delta z = 1 \) m thus results in 3200 Hz data generation rate. Figures 5 and 6 show amplitude and Doppler frequency shift of the modeled complex signals sampled at 50 Hz rate. Model refractivity profiles A and B result in rather smooth observational data except for some oscillations of amplitude and Doppler induced by the superrefraction layer (profile B). The structure of the RO signals modeled with the use of the radiosonde refractivity profiles 1, 2, and 3 is more complicated. At first we note that the complicated refractivity structure results in propagation of radio waves down to significantly lower observational altitudes than in the case of a smooth refractivity profile A. Mean amplitude is decreasing down to approximately -150 km altitude and remains about constant below. Both amplitude and phase undergo strong fluctuation. The structure of the fluctuation is different above and below approximately -10 km observational altitude, i.e., as the signal descends into the moist troposphere below \( \sim 6 \) km (the
The upper boundary of the moisture-induced irregularities in refractivity is visible in the radiosonde profiles in Figure 3). At lower altitudes, i.e., after multipath propagation through the moist troposphere, one cannot explicitly see mapping of the refractivity irregularities into the irregularities of amplitude and Doppler (which could be seen in the case of a single-path propagation). Here the whole observed complex signal may be treated as a radio hologram which contains information about the whole refractivity profile.

Figure 7 shows the results of spectral analysis of the modeled RO signals. For the spectral analysis we use window $\Delta z = 4.096$ km, or $\Delta t = \Delta z/v_\perp = 1.28$ s, which allows the spectral resolution $1/\Delta t \approx 0.78$ Hz. Since during 1.28 s the mean Doppler has significant lapse, at first the signal was down-converted within each window with the use of the linear least squares fit of the Doppler. Then the spectral amplitudes $[(\text{Re } c_m)^2 + (\text{Im } c_m)^2]^{1/2}$ were calculated as functions of frequencies $f_m = m/\Delta t$. Figures 7a-7f show the spectra of signal 1 calculated at different observational altitudes (indicated in each graph). As seen, the double-sided spectral bandwidth of the signal above the moist troposphere (+20 km observational altitude) is within $\sim 10$ Hz, while after propagation through the moist troposphere it is $\sim 50$ Hz. The structure of the spectra is rather complicated, with multiple local maxima corresponding to multiple tones (rays). Figures 7g-7i show mean spectra, i.e., obtained by averaging all spectra for all three signals calculated within the observational altitude range as indicated in each graph (each individual spectrum was calculated with the use of the window $\Delta z$ defined above). It can be noted that the shape of the RO signal spectra is different at different altitudes. It is close to symmetric in Figure 7g, and it shows significant asymmetry in Figure 7i. This asymmetry can be explained by the impact of the Earth’s surface. For the observational altitudes correspond-
ing to propagation of the main RO tone (ray) well above the Earth's surface the auxiliary tones (rays) are arriving from both above and below the main one and thus have both positive and negative frequencies with respect to the main tone. For low enough observational altitudes the main tone (ray) is "sliding" along the surface, and auxiliary tones (rays) are arriving from above, which explains the asymmetry in the spectrum.

Deep dips in amplitude after propagation of RO signals through the moist troposphere, as seen in Figure 5, may result in the loss of lock when tracking such signals with PLL. Another parameter which is crucial for the PLL tracking is phase acceleration. According to Kaplan [1996], for stable operation of the PLL in a generic GPS receiver, the phase acceleration must not exceed 6 g, i.e., ~300 Hz/s. Figure 8 shows the finite difference estimate of the phase acceleration, \( (\phi_{n+1} - 2\phi_n + \phi_{n-1})/(2\pi\Delta t^2) \), where \( \Delta t = 1/50 \) s, calculated for signal 1. As seen, the random phase acceleration after propagation through the moist troposphere is significantly larger than that allowed for the stable operation of a generic GPS receiver. This indicates that a tracking technique which is more stable with respect to random phase accelerations and to dips in amplitude than PLL has to be applied for RO signals that propagate through the moist troposphere. It may be open loop (OL) tracking, i.e., sampling of the complex signal after its in-real-time down-conversion in the receiver with the use of the Doppler model, which is based on predicted orbits of GPS and LEO satellites and refractivity climatology [Sokolovsky, this issue].

6. Inversions of Radio Occultation Signals

Inversions of RO signals include reconstruction of the bending angle as a function of impact parameter (under the assumption of spherical symmetry of refractivity). Then the bending angle can be either directly assimilated by NWP models [Kuo et al., 2000b] or used for the reconstruction of refractivity as a function of altitude by means of Abel inversion [Eshleman, 1973]. Routinely, the bending angle and impact parameter are calculated from the Doppler
Figure 7. Spectra of RO signals at different observational altitudes. (a-f) Individual spectra. (g-i) Averaged spectra.

frequency shift of the received RO signal (i.e., basically from the slope of the phase front) under the assumption of single-ray propagation [Vorob’ev and Krasil’nikova, 1994]. In the presence of multipath propagation this technique results in errors which may be rather large in the moist tropical troposphere. At present, two radio holographic methods solving for the multipath propagation (which had been previously applied for planetary atmospheres) are being tested for the Earth’s atmosphere [Karayel and Hinson, 1997; Gorbunov and Gurvich, 1998; Mortensen and Hoeg, 1998; Hocke et al., 1999; Gorbunov et al., 2000]. Both methods have been tested either with the use of some atmospheric models (which do not reproduce well enough the complicated vertical structure of refractivity typical for the moist troposphere) or with GPS/MET observational data (where true refractivity is not known exactly). Here we test these methods with the use of the modeled RO signals.

The first method uses back propagation (BP) of the received complex electromagnetic signal from the observational trajectory back to the atmosphere (i.e., solution of Helmholtz equation in a vacuum for a given boundary condition). For a straight line observational trajectory this could be done by expansion of the complex electromagnetic field in a series of plane waves, as discussed in section 4. For a curved trajectory (i.e., for observations from LEO) the integral form of the solution of Helmholtz equation has to be used [Born and Wolf, 1964]. Back propagation in a vacuum is equivalent to straight line continuation of rays (i.e., it preserves their impact parameters). Thus, even though the back propagated field is not equal to true electromagnetic field in the atmosphere, it provides the same bending angle as a function of impact parameter. This allows one to calculate this function on some auxiliary trajectory closer to the Earth’s limb where multipath is expected not to be present (or, at least, substantially reduced). The BP method, which originally had been applied for planetary atmospheres [Marouf, et al., 1986], has been tested for the Earth atmosphere [Karayel and Hinson, 1997; Gorbunov and Gurvich, 1998; Mortensen
This method has been demonstrated to perform well for the signals simulated with the use of NWP models [Gorbunov and Gurvich, 1998]. It was also used for routine processing of the GPS/MET data [Rocken et al., 1997]. A key point of the BP method is finding the position of the auxiliary trajectory in a one-ray region. If such position is found, then the BP method can provide very high resolution (theoretically up to the wavelength). However, there is no a priori criterion to find the position of the auxiliary trajectory in a one-ray region (under certain conditions, such as superrefraction, it may not be possible at all [Gorbunov et al., 2000]). This is the main problem of the BP method.

The second method, radio optical, which we call here the sliding spectral (SS) method, uses spectral analysis of the received complex signal within some small apertures. Then the local spectral maxima within each aperture are associated with multiple plane waves, i.e., rays, arriving at that aperture. The frequency of each spectral maximum and the position of the center of the aperture define the bending angle and the impact parameter of each ray. The SS method originally also had been applied for planetary atmospheres [Lindal et al., 1987] and, recently, for the Earth’s atmosphere [Hocke et al., 1999; Gorbunov et al., 2000]. A key point of the SS method is identification and selection of local maxima in the spectrum of the RO signal. For complicated spectra (like those shown in Figure 7) this introduces a serious problem (especially for an automated processing) and may allow different approaches. Hocke et al. [1999] used the multiple signal classification (MUSIC) technique and tested the SS method by processing several GPS/MET occultations. Gorbunov et al. [2000] used Fourier analysis and applied the SS method to RO signals simulated with the use of NWP models. However, a technique to identify and select the local spectral maxima was not introduced. None of the papers introduces a technique that allows for an automated (without supervision) calculation of bending angle as an unambiguous function of impact parameter based on spectral analysis of the complex RO signal.

Here we introduce a simple technique for the SS method which does not require any identification and selection of local spectral maxima (and thus may be used for automated data processing). Instead, the whole spectral content of the signal within each aperture is utilized, and each ray is weighted proportional to the spectral power. On output, bending angle is reconstructed as an unambiguous function of impact parameter.

We apply the SS method for the straight line observational trajectory which is normal to the direction of the incident plane wave, as shown in Figure 9 (generalization for a point source at a finite distance (GPS) and an arbitrary observational trajectory (LEO) does not introduce any problem). At first the complex signal received on the observational trajectory, \( u(z) = a(z) \exp \{iks(z)\} \), is downconverted with the use of some phase model \( s_{\text{mod}}(z) \) which removes the main trend in phase, \( v(z) = a(z) \exp \{iks(z) - s_{\text{mod}}(z)\} \). For the phase model \( s_{\text{mod}}(z) \) we use cubic spline regression of the continu-

![Figure 8. Phase acceleration of RO signal 1 sampled at 50 Hz rate (dashed lines show ±300 Hz/s allowed for stable operation of a generic GPS receiver).](image)

![Figure 9. Layout of the reconstruction of bending angles from the complex RO signal by SS technique.](image)
ous (accumulated) phase \( s(z) \) for the whole occultation. Then the signal \( u(z) \) is considered within some small aperture \( \Delta z \) centered at \( z_c \), and its complex Fourier transform is calculated within that aperture,

\[
c_m = (2\pi)^{-1} \int_{z_c-\Delta z/2}^{z_c+\Delta z/2} u(z) \exp(-2\pi imz/\Delta z)dz,
\]  

where \( m/\Delta z \) is spatial frequency. With the use of the discrete Fourier transform \((-M_1/2 \leq m \leq M_1/2)\), where \( M_1 \) is the number of data in the window \( \Delta z \) the signal \( u(z) \) is represented within the aperture \( \Delta z \) through the sum of \( M_1 \) harmonics, each having the spatial frequency \( \omega_m = (ms\text{mod}/\Delta z)|_{z=z_c}/\lambda + m/\Delta z \) and the power \( |c_m|^2 \). Each harmonic is associated with the ray arriving at the aperture at angle \( \epsilon_m \) and thus having impact parameter \( \rho_m \), as shown in Figure 9,

\[
\begin{align*}
\epsilon_m &= \sin^{-1}(\omega_m/\lambda), \\
\rho_m &= (R_E + z_c + l \tan \epsilon_m) \cos \epsilon_m.
\end{align*}
\]

By sliding the aperture \( \Delta z \) along the observational trajectory a large array of overlapping rays \( \{\rho_j, \epsilon_j, |c_j|^2\} \) is collected, where \( 1 \leq j \leq M_1(M - M_1) \), where \( M \) is the number of data in the whole processed RO signal. Then these rays are sorted according to increasing impact parameter \( \rho_j \). The function \( \epsilon_k = \epsilon(\rho_k) \) is then obtained from the sorted array \( \{\rho_j, \epsilon_j, |c_j|^2\} \) by sliding window averaging of both \( \{\epsilon_j\} \) and \( \{\rho_j\} \) with the weighting factor \( |c_j|^2 \),

\[
\begin{align*}
\alpha_k &= \frac{1}{w_k} \sum_{j=k-K/2}^{k+K/2} \epsilon_j |c_j|^2, \\
\rho_k &= \frac{1}{w_k} \sum_{j=k-K/2}^{k+K/2} \rho_j |c_j|^2, \\
w_k &= \sum_{j=k-K/2}^{k+K/2} |c_j|^2.
\end{align*}
\]

In the bending angle thus calculated as an unambiguous function of impact parameter \( \alpha(\rho) \) the contribution of each spectral component (a priori associated with a ray) is proportional to its power, and thus no identification and selection of local spectral maxima is necessary. Testing this technique shows that the application of the sliding aperture for the spectral analysis of RO signal allows for noticeable improvement of resolution as compared to binned aperture analysis. The large array of rays collected after sliding aperture analysis makes the retrieved \( \alpha_k = \alpha(\rho_k) \) not too sensitive to the size of the window used for sliding averaging in (10) within a rather large range of \( K \), and it even allows for binned averaging without noticeable degradation of the results. For the sliding aperture \( \Delta z \) we tested 1.024, 2.048, and 4.096 km and found that the results are not very different, but on average, \( \Delta z = 2.048 \) km provides better resolution. The simplicity of the discussed technique (which is contained in taking into account the whole spectral content of a signal) comes at the expense of the vertical resolution. Hypothetically, the resolution could be better would it be possible to sort out the "false" spectral maxima [Gorbunov et al., 2000].

For comparisons with the bending angles retrieved by BP and SS methods we use the bending angle function \( \alpha(\rho) \) calculated from the refractive index profile \( n(h) = 1 + 10^{-6}N(h) \) by means of geometric optics (GO),

\[
\alpha(\rho) = -2\rho \int_\rho^{\infty} n^{-1} dn/dx(x^2 - \rho^2)^{-1/2} dx,
\]

where \( x = r n(r) \) is refractive radius, \( r = R_E + h \). Under normal conditions the function \( x(r) \) is monotonic, \( dx/dr > 0 \). However, large enough refractivity gradients may result in superrefraction, i.e., \( dx/dr < 0 \). Under the superrefraction conditions, rays with certain altitudes of tangent point are internal (trapped). The apogees of the trapped rays are inside the superrefraction layer, and their perigees (which have the same \( x \) as the apogees) are below the layer. Two external (untrapped) rays whose altitudes of tangent points are infinitely close to the highest and the lowest altitudes of the internal rays have the same \( \rho \) and infinitely large \( \alpha \). Thus \( \alpha(\rho) \), formally calculated for the external rays, has singularity, but it has no gap in \( \rho \) (there is a gap in the altitude of the tangent point for the external rays). Abel inversion [Eshleman, 1973] assumes a monotone function \( x(r) \). Its formal application to the function \( \alpha(\rho) \) calculated for the external rays in the case of superrefraction results in negative errors in retrieved refractivity inside and below the superrefraction layer. Multiple layers result in cumulative error. The problem of superrefraction and related inversion errors is discussed by Kursinski et al. [1997] (however, the suggestion to derive the structure within the superrefraction in-
tervals via interpolation is not clear). Identification of the superrefraction layers by infinitely large spikes of the bending angle is possible only theoretically, in GO approximation. Although GO is not strictly applicable for refractivity profiles such as profiles 1, 2, 3, and B at the L1 GPS wavelength used (even without the superrefraction, the vertical scales of the refractivity irregularities are smaller than the Fresnel zone), we still use the comparison of BP and SS $\alpha(\rho)$ to GO $\alpha(\rho)$ for preliminary evaluation of the resolution of the radio holographic methods in the bending angle space before reconstruction of the refractivity.

At first we applied the BP and SS methods for inversions of the signals obtained with the use of the model refractivity profiles A and B shown in Figure 3 (this is necessary for validation of the forward and inverse techniques). For the exponential model (model A), BP and SS $\alpha(\rho)$ are indiscernible from GO $\alpha(\rho)$, as shown in the left graph in Figure 10. The middle graph shows $\alpha(\rho)$ for the superrefraction layer (model B) reconstructed with the BP method, for three positions of the auxiliary trajectory. It is difficult to make a choice for the optimal position of the auxiliary trajectory (in the case of superrefraction it is strictly not possible to position the auxiliary trajectory in a single-ray region). The right graph shows $\alpha(\rho)$ for the superrefraction layer (model B) reconstructed with the SS method. Both BP and SS methods provide feasible results above and below the superrefraction layer and introduce some errors inside the layer.

Next we applied the BP and SS methods for inversions of the signals modeled with the use of high-resolution radiosonde refractivity profiles 1, 2, and 3 shown in Figure 3. For the BP method it appears to be a problem to find the position of the straight line auxiliary trajectory, normal to the direction of the initial propagation, which allows one to obtain an unambiguous (or at least feasible) function $\alpha(\rho)$. Figure 11 shows three functions $\alpha(\rho)$ obtained after back propagation of the complex signal modeled from refractivity profile 1 to auxiliary trajectory at a distance $l = 100, 150, \text{and } 200 \text{ km from the limb}$. In each case, $\alpha(\rho)$ contains significant unresolved ambiguities at different altitudes. At some altitudes where BP $\alpha(\rho)$ is unambiguous, it fairly well coincides with GO $\alpha(\rho)$; at other altitudes it does not. In particular, it is not possible to reconstruct the deep dip in GO $\alpha(\rho)$ at $\rho \approx 6373.6 \text{ km}$ for any position of the auxiliary trajectory. Some criteria for finding an optimal trajectory (which may be curved and rather complicated) are outlined by Gorbatov et al. [2000], however, they have not been demonstrated yet. Sometimes a small shift in the position of the auxiliary trajectory, $\sim 5-10 \text{ km}$, results in significant change of the reconstructed oscillations of BP $\alpha(\rho)$. This

![Figure 10](image)

**Figure 10.** Bending angles reconstructed by BP and SS techniques for refractivity models A and B (solid curves); GO bending angles (dashed curves). Here, $l$ is the distance from the limb to the auxiliary trajectory.
sensitivity to the position of the auxiliary trajectory was also found when processing GPS/MET tropical occultations with the use of the BP method. This indicates that high accuracy and resolution of the BP method (which may be fully realized in the case of propagation through the single phase screen) are questionable in the case of the spherically symmetric atmosphere with the vertical structure reproduced by high-resolution radiosondes. It may be possible that the BP method performs better for the real troposphere which is not strictly spherically symmetric, and thus the effect of the superrefraction may not be so exposed as in this study.

Figure 12 shows the results of the reconstruction of $\alpha(\rho)$ by the SS method for refractivity profiles 1, 2, and 3. As seen, in this case, SS $\alpha(\rho)$ reproduces the structure of GO $\alpha(\rho)$ fairly well at scales of $\sim0.5$ km and larger but fails to resolve the smaller-scale structure. The SS $\alpha(\rho)$ shown in Figure 12 were used for reconstruction of the refractivity by Abel inversion,

$$n(x) = \exp \left[ \frac{-1}{\pi} \int_{x}^{\infty} \frac{\alpha(\rho)(\rho^2 - x^2)^{-1/2}}{d\rho} \right]. \quad (12)$$

The reconstructed profiles $N(h)$, where $h = x/n - R_E$, are shown by solid curves in the left graph in Figure 13 ("true" profiles are shown by dashed curves for comparison). As seen, the structure of the refractivity with scales of $\sim0.5$ km and larger is reconstructed fairly well. The right graph shows the retrieval errors. The largest errors, including some mean error, are below $\sim2-4$ km. We believe that the mean error (bias) is introduced by the multiple superrefraction layers (a negative bias of about the same magnitude can be obtained after Abel inversion of GO $\alpha(\rho)$).

For validation of the effect of superrefraction on the bias after inversions we performed the following simulations: We modeled RO signals by replacing the spherically symmetric atmosphere with the single phase screen. For a refractivity profile such as shown in Figure 3 this results in multipath propagation as severe as that in the case of multiple phase screens, and the structure of RO signal is as complicated as that in Figures 5 and 6. However, in the case of the single phase screen there is no such phenomenon as superrefraction (i.e., ray trapping). For the single phase screen the BP method provides exact solution of the inverse diffraction problem, i.e., reconstruction of the phase on the screen, which is trivial. The SS method resulted in random inversion errors of the reconstructed phase on the screen of about the same fractional magnitude as the errors of the reconstructed refractivity after multiple-phase-screen propagation. However, in the case of the single phase screen there was no evidence of the mean error (bias). Also, we modeled RO signals with the use of multiple phase screens for random isotropic irreg-
ularities of refractivity whose magnitude was large enough to provide strong fluctuation of amplitude and Doppler. In that case the inversion with the use of the SS method resulted in random retrieval errors without evidence of bias. Thus the superrefraction introduces a problem for RO technique in the lower troposphere.

Bias error (if it cannot be predicted and corrected) introduces a larger problem for meteorology and climatology than random error does. The bias which can be seen in Figure 13 is clear overestimation of the RO inversion bias due to the superrefraction. This is due to the assumption of the spherical symmetry in refractivity applied in the forward modeling. This assumption results in a large number of thin superrefraction layers associated with the small-scale irregularities in vertical refractivity profile, which is not fully realistic. For more realistic evaluation of the inversion errors in the moist troposphere it is necessary to perform simulations with more realistic 3-D (2-D) refractivity models. It must be noted that the bias in Figure 13 can not fully explain the negative bias in the GPS/MET data, which is larger and which, in most part, is believed to be related to the signal-tracking technique (since its magnitude substantially depends on the receiver tracking firmware, as discussed in section 1).

As was mentioned, detection of the superrefraction from RO signals is a difficult problem. It may be possible that concurrent processing of RO signals by BP and SS methods can be used as a quality indicator of an occultation. Significant disagreement between the BP and SS bending angles may indicate superrefraction and, as a result, negative bias in refractivity. On the basis of the disagreement the occultation may be qualified as "low quality" or discarded. This approach must be tested by simulations and by processing of observation data.

As was noted, diffraction by laminated irregularities of refractivity results in propagation of radio waves down to substantially lower observation altitudes than in the case of a smooth refractivity profile. It appears that the information contained in RO signal at those altitudes is important for radio holographic inversions. The $\alpha(r)$ and $N(h)$ in Figures 12 and 13 were reconstructed by SS method with the use of the modeled RO signal down to -160 km observation altitude (using the signal down to lower altitudes did not noticeably change the results). However, if the RO signal is used down to -80 km observation altitude only (which is about the mean altitude where the receiver stopped tracking because of "loss of lock" during "prime time" 2 in GPS/MET), then the negative bias is larger than that in Figure 13. Thus tracking RO signals in the moist troposphere is important down to low enough observation altitudes to minimize the inversion errors. This can be done by means of OL technique [Sokolovskiy, this issue].

Figure 12. Bending angles reconstructed by SS technique for refractivity profiles 1, 2, and 3 (solid curves). GO bending angles (dashed curves).
7. Conclusions

Forward modeling of RO signals and their radio holographic inversions performed with the spherically symmetric refractivity modeled from high-resolution tropical radiosondes lead to the following conclusions.

1. The phase and amplitude of RO signal observed in LEO undergo strong fluctuation as the signal descends into the moist lower troposphere with a complicated structure of refractivity. The bandwidth of the signal increases from ~10 Hz above the moist troposphere to ~50 Hz below. The random phase acceleration of the signal may be as large as ~1000 Hz/s, i.e., much larger than allowed for stable signal tracking by a conventional GPS receiver. The signal amplitude has deep dips which may cause loss of lock when tracking such signals with a PLL.

2. Inversions of the simulated tropospheric RO signals by use of the BP method are faced with the serious problem of finding the position of the auxiliary trajectory to obtain bending angle as an unambiguous function of the impact parameter. For the SS method the introduced technique, which utilizes the whole spectral content of the RO signal, allows for stable reconstruction of the bending angles and refractivity with vertical resolution of ~0.5 km in automated processing. Retrieval errors include random errors due to the insufficient resolution of the introduced inversion technique and some negative bias below ~2-4 km which is believed to be related to the superrefraction layers. Treatment of all small-scale refractivity structures reproduced by the high-resolution radiosondes as spherically symmetric results in clear overestimation of the bias error. For more accurate error characterization of the RO technique in the lower troposphere, simulations with more realistic 2-D (3-D) tropospheric models must be performed.

3. The small-scale laminated structure of refractivity in the moist troposphere causes propagation of RO signals down to significantly lower observational altitudes than do smooth refractivity profiles. The information content of the RO signal (radio hologram) at those low altitudes is important, and failure to include it into radio holographic inversions may cause additional errors in the reconstructed refractivity, including bias. Thus tracking RO signals down to sufficiently low observational altitudes (corresponding to about -150 km altitude of the tangent point of straight line GPS-LEO for 700-800 km LEO altitude) is very important. A tracking technique which provides stable performance under conditions
of strong amplitude and phase fluctuation and low mean signal power (OL tracking) has to be applied in the troposphere, especially in the tropics. The receivers for future RO missions must be tested with simulated signals similar to those used in this study.

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