Postprocessing of L1 GPS radio occultation signals recorded in open-loop mode

S. Sokolovskiy,1 C. Rocken,1 W. Schreiner,1 D. Hunt,1 and J. Johnson1,2

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GPS Radio Occultation (RO) profiling from low-Earth orbiting satellites is operationally used for numerical weather forecasting and is starting to be used for climate studies. Obtaining high-quality observations near the surface requires recording of RO signals in model-aided open loop (OL) mode by the GPS receiver. Postprocessing of the OL RO signals is different from that of the signals recorded in traditional phase-locked loop (PLL) mode. It requires modeling of the signal frequency for connection of the phase between samples and removal of the GPS navigation data modulation (NDM). It is important that the postprocessing does not introduce errors (biases) in the connected phase. This paper describes the postprocessing of the OL RO signals which does not depend on the receiver model. The postprocessing includes: modeling of the RO signal frequency from refractivity climatology and the subsequent adjustment of this model by use of feedback which makes the postprocessing model-independent; internal (as in PLL) removal of NDM and the use of externally recorded NDM bit sequence. Statistical comparison of the refractivity inversion results demonstrates that external demodulation of RO signals reduces the inversion bias in the tropical lower troposphere by about 0.5%.


1. Introduction

Radio occultation (RO) by use of the Global Positioning System (GPS) and low Earth orbiting (LEO) satellites has proven to be an accurate technique for remote sensing of the Earth's atmosphere [Ware et al., 1996; Kursinski et al., 1997; Rocken et al., 1997; Wickert et al., 2001; Hajj et al., 2004; Kuo et al., 2004; Schreiner et al., 2007; Anthes et al., 2008; Foelsche et al., 2008]. The observables are the phase and the amplitude of RO signals. They are converted to vertical profiles of ray bending angles and refractivity. The refractivity in the neutral atmosphere is locally related to pressure, temperature and partial pressure of water vapor [Thayer, 1974].

Both the bending angle and the refractivity can be assimilated by atmospheric models [Eyre, 1994; Poli, 2006; Cucurull et al., 2007; Ringer and Healy, 2008].

Since the collection of the first GPS RO data in 1995 [Ware et al., 1996] it was found that RO encounters problems in the moist lower troposphere resulting in significant refractivity inversion errors that include biases [Rocken et al., 1997]. Later, it became obvious that these errors are related to the errors of signal tracking by means of the phase-locked loop (PLL) [Sokolovskiy, 2001b; Ao et al., 2003; Beyerle et al., 2003; Beyerle et al., 2006] and to propagation effects such as superrefraction [Sokolovskiy, 2003; Xie et al., 2006; Ao, 2007]. This study is related to resolving the first problem, elimination of the tracking errors in the lower troposphere by applying an alternative tracking technique instead of the PLL.

The complicated structure of refractivity in the moist lower troposphere (related mainly to the complicated structure of humidity) causes multipath propagation, which results in strong random phase and amplitude modulation of the RO signals. The diffracted RO signals propagate deep behind the Earth’s limb where they have to be recorded under conditions of low signal-to-noise ratio (SNR). Strong vertical refractivity gradients on top of the moist atmospheric boundary layer result in significant fading of the RO signal amplitude due to defocusing followed by an increase in amplitude at lower heights [Hajj et al., 2004]. An added complication is posed by the 50 Hz navigation data modulation (NDM) [0, π] imposed on the phase of GPS signals, which has to be
distinguished from the atmosphere-induced modulation (AIM) in order to be removed from the RO signals.

[5] The PLL [Beyerle et al., 2006], which is an optimal tracking technique for single-tone (narrowband) signals under sufficient SNR, has been routinely used in GPS receivers. The PLL extracts the signal phase in real time and projects the expected phase ahead. The projected phase model is used for down conversion of the signal, i.e., shifting its mean frequency close to zero. The down-converted signal is subject to low-pass filtering (integration). Then the residual (unmodeled) phase is extracted and used for updating the phase model for the next sampling interval. Concurrently with the extraction of the phase, the NDM is removed under the assumption that the phase lapse between 50 Hz samples (typical GPS receiver internal sampling rate) induced by AIM is smaller than \( \pi/2 \) (half a carrier signal wavelength). Thus, with the removal of the NDM, the phase can be extracted in only 2 quadrants. When the phase lapse due to AIM is larger than \( \pi/2 \) or SNR is low, the errors of the phase extraction increase (in particular, these errors include the errors of removal of the NDM, half-cycle slips). As a result, the projection of the phase ahead becomes less accurate. When the difference between the projected and the true frequencies becomes larger than half of the filter bandwidth, the amplitude reduces to noise level, further tracking becomes unstable and, after some time, the receiver declares loss of lock. The results of PLL tracking of RO signals in the moist lower troposphere are sensitive to tunable loop parameters [Sokolovskiy, 2001b; Beyerle et al., 2006]. A serious limitation for the PLL tracking of rising occultations. Sufficient SNR and substantial time, required to lock on the signal emerging from behind Earth’s limb, would result in loss of the lowermost part of the RO signal.

[6] Raw sampling of the complex RO signal, i.e., open loop (OL) tracking, is free of the problems discussed above and had previously been applied for RO studies of the planetary atmospheres. The sampling rates applied in those studies, typically, were much higher than required to transfer the information contained in RO signals (for example, 50 and 150 kHz at S and X bands [Lindal et al., 1983]). Application of high sampling rates simplifies signal processing but is neither needed nor efficient for regular GPS RO remote sensing of the Earth’s atmosphere because of high data downlink bandwidth requirements. In order to reduce the sampling frequency to the minimal Nyquist-required rate, a novel model-aided OL tracking approach was developed.

[7] The principles of the model-aided OL tracking of RO signals were introduced by Sokolovskiy [2001b]. They essentially use the fact that the Doppler frequency shift of the RO signal is related to the arrival angle of the ray at the receiving antenna. This arrival angle is influenced by Earth’s atmosphere and, very importantly, its variations decrease with an increase of the distance from the receiver to the limb. For RO observations from LEO, because of large LEO-to-limb separation, the mean Doppler frequency shift caused by all of Earth’s atmospheric conditions can be predicted (modeled) with very good accuracy to within 10–15 Hz, without feedback from the RO signal.

[8] In a receiver operating in OL mode, the complex RO signal is down-converted with the frequency model based on a real-time navigation position/velocity/clock solution and a simple bending angle model, then subjected to low-pass (noise) filtering and sampling. According to the Nyquist theorem, the sampling frequency shall not be smaller than the double-sided spread of the signal spectrum (which, in most cases, does not exceed 50 Hz for RO signals recorded from LEO [Sokolovskiy, 2001a]). Theoretically, the signal can be sampled at the Nyquist-required rate without any down conversion but with the possibility of spectral aliasing. Then this aliasing can be eliminated by down conversion of the sampled signal in postprocessing. However, practically, this would result in incoherent summation of noise aliased into the sampling band, and thus low SNR. Thus, the purpose of the real-time down conversion of the RO signal in the receiver is entirely noise filtering prior to sampling.

[9] The frequency mismodeling in the receiver does not change the phase of the complex samples (model + residual), but may reduce the SNR. It is important that the signal spectrum remains within the main lobe of the filter response function. In postprocessing, the signal, considered as the sequence of complex samples, is down-converted with a more accurate frequency model than in the receiver. The purpose of this down conversion is to remove the NDM and connect the phase between samples.

[10] The frequency mismodeling in postprocessing, also, does not change the phase of the complex samples, but it may introduce additional errors in the connected phase, i.e., half and full cycle slips. A frequency model based on the postprocessed navigation solution and the bending angle climatology is used only as the first guess. Then this model is further adjusted by determining the mean frequency of the RO signal reconstructed with this model and shifting the mean frequency to zero (this feedback from the whole reconstructed RO signal is different from recursive feedback in real time in PLL). This adjustment makes the results of the postprocessing of OL RO signals independent of the first guess model. This is important, especially, for climate applications.

[11] Six COSMIC (Constellation Observing System for Meteorology, Ionosphere and Climate) satellites launched on 15 April 2006 are recording L1 GPS RO signals in OL mode in the lower troposphere (overview of the first results from COSMIC can be found in the
work of Anthes et al. [2008]). In this study we apply the principles of OL RO signal processing for inversions of the COSMIC OL RO signals at the COSMIC Data Analysis and Archive Center (CDAAC) at the University Corporation for Atmospheric Research (UCAR). We discuss the use of the two frequency models (first guess and adjusted with the use of feedback) in postprocessing of OL RO signals. We also discuss two methods (internal and external) for removal of the NDM [Sokolovskiy et al., 2006]. We demonstrate the difference between the results of these different postprocessing modes based on statistical comparison of the retrieved refractivities in the tropical lower troposphere. Section 2 discusses the processing of OL RO signals. Section 3 demonstrates individual examples of postprocessing OL RO signals. Section 4 discusses the adjustment of the postprocessing model by use of the feedback from the RO signal and compares the inversion results with and without adjustment of the model and with external or internal removal of NDM. Section 5 presents the statistical evaluation of the differences in inversion results obtained with different processing modes. Section 6 summarizes the study.

2. Processing of the OL RO Signals

2.1. Recording of the OL RO Signals

The COSMIC RO receiver firmware was developed by Jet Propulsion Laboratory. Here, without entering into details, we introduce only those basic principles of signal processing in a RO receiver operating in the model-aided OL mode that are necessary for understanding the postprocessing of the OL RO signals.

A RO receiver operating in the model aided OL mode down converts the incoming complex RO signal (represented by I and Q signals) $u_{\text{in}}(t) = A_{\text{in}}(t) \exp[i\Phi_{\text{rcm}}(t)]$ by use of the phase model $\Phi_{\text{rcm}}(t)$ calculated in real time in the receiver without any feedback from the RO signal

$$u_{\text{down}}(t) = A_{\text{in}}(t) \exp[i\Phi_{\text{rcm}}(t) - i\Phi_{\text{in}}(t)]$$

where $t$ is time. Assuming a reasonably accurate phase model $\Phi_{\text{rcm}}(t)$ the signal $u_{\text{down}}(t)$ has low mean frequency. The down-converted signal is subjected to low-pass filtering by integration of I and Q over the intervals $\Delta t$ that may be equal to (or smaller than) output sampling intervals

$$I_{\text{lpf}}(t) = \Delta t^{-1} \int_{t}^{t+\Delta t} \text{Re}[u_{\text{down}}(t')] dt'$$

$$Q_{\text{lpf}}(t) = \Delta t^{-1} \int_{t}^{t+\Delta t} \text{Im}[u_{\text{down}}(t')] dt'$$

The complex signal on output of the receiver is:

$$u_{\text{out}}(t) = I_{\text{out}}(t) + iQ_{\text{out}}(t) = A_{\text{out}}(t) \exp[i\Phi_{\text{out}}(t)]$$

$$= A_{\text{out}}(t) \exp[i\Phi_{\text{rcm}}(t) + \Delta\Phi_{\text{out}}(t)]$$

where $\Delta\Phi_{\text{out}} = \arctan(Q_{\text{out}}/I_{\text{out}})$ (defined in 4 quadrants) and $A_{\text{out}} = \sqrt{I_{\text{out}}^2 + Q_{\text{out}}^2}$.

In the COSMIC receivers the signal $u_{\text{out}}(t)$ is sampled at 50 Hz rate. The phases $\Phi_{\text{in}}$ and $\Phi_{\text{out}}$ contain the [0, $\pi$] NDM. The integration intervals $\Delta t$ are aligned with the 20 ms NDM chips, which is achieved by accounting for the propagation time between GPS and LEO. This is common for both PLL and OL tracking. However, in PLL tracking, the NDM is removed in the receiver and the output phase-connected data are commonly interpolated to standard 20 ms receiver time tags. This interpolation is not possible in OL tracking, i.e., it would result in significant errors of $\Phi_{\text{out}}$ due to the unremoved NDM. Thus each OL sample is supplied with an individual receiver time tag different from the standard 20 ms receiver time tags. The OL time tags are adjusted in postprocessing by use of the postprocessed navigation solution that includes correction of the receiver clock offsets.

In addition to the frequency model, the receiver operating in OL mode is also modeling the pseudorange (geometric range plus the effect related to propagation in the atmosphere with account for LEO clock offset) in real time. Modeling of the pseudorange is necessary for aligning the C/A code replica with the RO signal in order to remove the C/A code modulation (to despread the spectrum) before integration and sampling (removal of the C/A code modulation in postprocessing would require sampling rate $\sim$10 MHz).

2.2. Signal-to-Noise Ratio of the OL RO Signals

Occultation antennas of the COSMIC receivers allow signal-to-noise ratio (SNR) (scaled to 1Hz bandwidth) of about 800 V/V for GPS transmitters observed in the antenna boresight direction. This SNR is achieved with optimal noise filtering resulting from modeling of the phase in the PLL and modeling of the pseudorange in the delay-locked loop (DLL) by use of the feedback from the signal (which is possible for a narrowband signal). In the OL mode the SNR, generally, is lower because neither frequency nor pseudorange are modeled as optimally as in stably operating PLL/DLL.

Figure 1 allows estimation of the loss of amplitude (SNR) due to errors in modeling of the frequency and pseudorange in the receiver. Figure 1a shows the response function of the low-pass filtering (18 ms integration of I and Q [Thomas, 1995]) used in COSMIC
receivers. For example, frequency mismodeling of about 20 Hz would result in reduction of the signal amplitude by about 20%. Figure 1b shows the autocorrelation function of the C/A code. For example, pseudorange mismodeling of about 100 m would result in reduction of the signal power by about 40% (amplitude by about 20%).

[18] On average, the accuracy of OL models in the COSMIC receivers is about 10–20 Hz for the frequency model and about 100 m for the pseudorange model (the tropospheric propagation is mainly responsible for the frequency mismodeling, while the ionospheric propagation is mainly responsible for the pseudorange mismodeling). Besides that, during some limited time intervals, the errors of the receiver models may be much larger due to certain firmware problems; such cases are considered outliers.

[19] In this study we represent RO signals and their models as functions of the height of the straight line (HSL) connecting GPS and LEO shown in Figure 2 (due to ray bending, the HSL is negative at the bottom of an occultation). The use of HSL instead of time allows for a more meaningful representation of a RO signal in terms of its penetration behind the atmosphere.

[20] Figure 3 shows examples of the differences between the frequency and pseudorange models calculated in real time in the COSMIC receivers and in postprocessing (Figures 3a, 3c, and 3e) and the corresponding L1 SNRs which are proportional to the amplitude (Figures 3b, 3d, and 3f). Figures 3a and 3b show examples of normal occultation with accurate receiver frequency and pseudorange models. Figures 3c–3f show examples of outliers when large errors of the receiver frequency and pseudorange models result in reduction of amplitude to noise level. In those cases the phase has no physical sense and it follows the model (with random noise) by

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**Figure 1.** (a) Frequency response function of the low-pass filter, 18 ms integration, and (b) C/A code autocorrelation function. The dashed lines indicate the amplitude loss due to frequency modeling error of 20 Hz (Figure 1a) and due to a pseudorange mode error of 100 m (Figure 1b).

**Figure 2.** Height of straight line (HSL) connecting GPS and LEO (is negative at the bottom of occultation due to large bending in the lower atmosphere).

**Figure 3.** (a, c, and e) Examples of the differences between receiver and postprocessing first guess frequency (solid lines) and range (dotted lines) models for three COSMIC occultations. (b, d, and f) The corresponding SNRs. First occultation (Figures 3a and 3b) is normal (accurate receiver models); second (Figures 3c and 3d) and third (Figures 3e and 3f) occultations are affected by large frequency and range receiver model errors resulting in significant reduction of SNR in RO signals.
resulting in inversion errors. Such outliers as shown in Figures 3c–3f are infrequent in COSMIC receivers; they are shown here solely to illustrate the importance of accurate frequency and pseudorange modeling in a RO receiver operating in model-aided OL mode. More common are very large but very short outliers (affecting one sample). Those outliers, though being short would result in significant inversion errors due to their magnitude. Replacement of the receiver phase model by the postprocessing model (outlier-free) substantially reduces the inversion errors in cases of any outliers and basically eliminates the inversion errors in cases of very short outliers.

2.3. Postprocessing of the OL RO Signals

[21] The main difference between processing the OL and PLL RO data is that in OL mode both the NDM (which manifests itself as half-cycle jumps) has to be removed and the phase has to be connected between samples (full cycle ambiguities removed) in postprocessing. Both of these functions are handled by the receiver in PLL tracking.

[22] In order to remove the NDM and connect the phase, the sampled signal must be down-converted to shift its mean frequency as close to zero as possible. Such down conversion is already done by use of the model \( \Phi_{\text{rem}}(t) \) running in the receiver. However, in postprocessing, generally, it is possible to model the mean RO signal frequency more accurately than in the receiver for several reasons: (1) the frequency model depends on the navigation solution which is more accurate in postprocessing than in real time; (2) the receiver model is sometimes affected by outliers; (3) in the COSMIC receivers, limited computational resources pose restrictions on application of more advanced bending angle algorithms; (4) in postprocessing, the first guess of the atmospheric state and the feedback from the whole recorded RO signal can be used for adjusting the model (see section 4). We note that some of the reasons listed above may be relaxed or eliminated in future receivers with more onboard computing power, and more precise real-time orbit determination capability.

[23] The postprocessing of COSMIC OL data, begins from the signal \( u_{\text{out}}(t) \) which is considered as the sequence of unconnected complex samples. It is important to recall that the receiver model \( \Phi_{\text{rem}} \) is not used in postprocessing, as was already mentioned in section 1 (the only role of the \( \Phi_{\text{rem}} \) is for noise filtering in the receiver). Nevertheless it is important that both the phase and the pseudorange receiver models should be part of the OL data that are sent to the postprocessing center for trouble shooting diagnostic purposes (as it follows from examples shown in Figure 3). In postprocessing, the signal \( u_{\text{out}}(t) \) is down-converted by use of the more accurate phase model \( \Phi_{\text{ppm}}(t) \)

The first guess of \( \Phi_{\text{ppm}}(t) \), can either be \( \Phi_{\text{ppm}}^0 \), which is based on the postprocessed navigation solution and the bending angle climate model (described in detail by Sokolovskiy [2001b]), or it can be the adjusted model \( \Phi_{\text{ppm}}^1 \), which is discussed in section 4. The bending angle climate model is derived from CIRA-86aQ climatology that includes water vapor [Kirchengast et al., 1999].

[24] The down-converted signal \( u(t) \) is a slowly rotating phasor. The rotation angle between \( i \) and \( i + 1 \) samples is \( \Delta \Phi = \arccos[(I_{i+1} + Q_{i+1})(I_i^2 + Q_i^2)^{-1/2} (I_{i+1} + Q_{i+1})^{-1/2}] \) where \( I = \text{Re}[u], Q = \text{Im}[u] \). When the mean signal frequency is zero then the rotation of the phasor is only caused by the AIM, NDM, and noise. When the effect of the AIM and the noise does not exceed \( \pi/2 \), then the NDM can be removed by inverting the phasor when \( \Delta \Phi > \pi/2 \), i.e.,

if \( (I_{i+1} + Q_{i+1}) < 0 \) then \( I_{i+1} \) and \( Q_{i+1} \) are inverted (multiplied by -1)

which is illustrated in Figure 4. We call this method (which is similar to that used in the PLL) internal removal of the NDM.

[25] Alternatively, the NDM can be removed, by applying a data bit sequence recorded by another ground-based (or spaceborne) GPS receiver that observes the occulting GPS at high enough elevation angles at the time of the occultation (to detect and record the NDM undisturbed by any significant amount of AIM). For collection of the NDM bit sequences, UCAR/COSMIC deployed a globally distributed network of ground-based GPS receivers located in the USA (Colorado), Germany,
Taiwan, New Zealand, South Africa, and Brazil. When all ground-based receivers operate, this network allows collection of the NDM bit sequences from all operational GPS satellites. The collected NDM bit sequences are supplied with the transmission time tags (RO signals are supplied with the reception time tags). To remove the NDM from the RO signals, the collected NDM bit sequences are aligned with the RO signals by accounting for their time tags and the propagation time between GPS and LEO. We call this method external removal of NDM.

[26] To connect the phase between the PLL and OL samples with internal removal of NDM, (5) is applied to all OL samples plus one adjoining PLL sample. After external removal of NDM, (5) is applied to the adjoining PLL and OL samples. If $\Delta \Phi > \pi/2$ then all OL samples are inverted.

[27] After internal or external removal of the NDM, the raw phase of the $i$th sample is extracted: $\Phi_i = \arctan(2) (Q_i, I_i)$ and then connected to the previous sample:

$$\Phi_i = \Phi_{i-1} + 0 \text{ or } + 2\pi \text{ or } -2\pi \text{ whichever minimizes the } |\Phi_i - \Phi_{i-1}| \quad (6)$$

Internal removal of NDM allows extraction of the phase in only 2 quadrants. In this case, the phasor rotation between samples due to AIM and noise larger than $\pi/2$ will be interpreted as a NDM phase jump, the phasor will be inverted, and this will result in error of $\pm \pi$ (half-cycle slip). External removal of NDM allows extraction of the phase in 4 quadrants. In this case it is possible to correctly connect the phase when the phasor rotation due to AIM and noise is less than $\pi$. A phasor rotation larger than $\pi$ will result in an error of $\pm 2\pi$ (cycle slip).

[28] The amplitude $A_{\text{out}}$ and the connected phase $\Phi_{\text{out}} = \Phi_{\text{ppm}} + \Phi$ are the output of the postprocessing of the OL RO signals and the input for calculation of the bending angle and refractivity; that is done in the same way for PLL and OL RO signals and is not discussed here.

3. Examples of the Postprocessing of COSMIC OL RO Signals

[29] The COSMIC RO receiver firmware records both L1 and L2 in the PLL mode above a certain HSL that is an adjustable parameter presently set to between $-10$ and $-20$ km. For LEO altitudes $400–800$ km this transition HSL corresponds to a ray tangent point (TP) height of about $8–10$ km. Below that height, the L1 GPS signal (modulated by C/A code) is recorded in the OL mode while L2 signal (modulated by P code) is not recorded (because modeling of pseudorange with accuracy better than the P code chip length $\sim 29.3$ m is not possible due to mainly lack of knowledge of the ionospheric delay). Setting and rising occultations are postprocessed in the same way.

[30] After NDM is removed and the phase is connected between samples, as discussed in section 2.3, the discrete 50-Hz RO signal is interpolated onto a uniform time grid for computational convenience. Since the changes of the sampling intervals due to the changes of the propagation delay are small, it is sufficient to simply assign a constant sampling interval $\Delta t = (t_2 - t_1)/(n - 1)$ to the RO signal, where $t_1$ and $t_2$ are time tags of the first and the last samples, $n$ is the total number of the samples. The inversion results with and without the interpolation (by assigning constant $\Delta t$) are indiscernible.

[31] Figures 5 and 6 show L1 RO signals for a high-latitude (Figure 5) and a low-latitude (Figure 6) COSMIC occultations. Figures 5a and 6a show amplitudes (SNR). Blue vertical lines show the transitions between PLL and OL tracking. It is seen that the high-latitude RO signal descends into noise more abruptly and at higher HSL than the low-latitude RO signal. For the low-latitude occultation, fading of amplitude at HSL $\sim -50$ km and emerging at HSL $\sim -60$ km corresponds to the sharp top of the moist atmospheric boundary layer. In postprocessing of the RO signals at CDAAC, the noise level of the L1 signal is determined by averaging SNR over the last three seconds at the bottom of an occultation. The signal is used for inversion above the lowest HSL where the SNR, smoothed with 1 s window, exceeds the noise level by 50%. These heights are shown by red vertical lines in Figures 5a and 6a. This approach is ad hoc and further research is needed to optimally determine the HSL that is used to truncate the RO signal.

[32] Figures 5b and 5c and Figures 6b and 6c show magnitudes of the phasor rotation $\Delta \Phi$ between 50 Hz samples after down conversion of the signal $u(t)$ with the models $\Phi_{\text{ppm}}^0$ and $\Phi_{\text{ppm}}^1$ (the PLL portions of RO signals are represented in the same way as the OL portions). In the PLL portions of RO signals, where NDM is removed by the receiver, the rotation of the phasor is caused only by the AIM plus noise. In the OL portion of the high-latitude occultation, where AIM is small, it is possible to clearly distinguish the NDM and AIM (samples are grouped around 0 and 0.5 cycles with a clear space between) with the use of both models $\Phi_{\text{ppm}}^0$ and $\Phi_{\text{ppm}}^1$. Thus for the high-latitude OL RO signal the NDM can be removed internally, as discussed in section 2.3. In the OL portion of the low-latitude occultation, the NDM can be better distinguished (the space between the samples grouped around 0 and 0.5 cycles is more clear) with the use of the model $\Phi_{\text{ppm}}^1$ rather than $\Phi_{\text{ppm}}^0$. Still, there is no clear space between the samples grouped around 0 and 0.5 cycles especially in the region of low SNR ($-55$ km $<$ HSL $< -30$ km). Thus for the low-latitude OL RO signal the NDM cannot be removed internally without errors (half-cycle slips) and external observations of the NDM are required.
Figures 5d and 6d show frequencies of the RO signals. Black lines show frequencies of the signals obtained by direct differentiation of the phase \( F_{\text{out}} \) on output from the receiver. These signals are not used; they contain unresolved full- and half-cycle ambiguities and are shown for illustration only. Red lines show frequencies of the RO signal after external removal of NDM and connection of the phase, as discussed in section 2.3 (these RO signals are ready for inversions). Separate windows show zoomed in sections of RO signals and the frequency models \( f_{\text{rcm}} \), \( f_{\text{ppm}}^0 \), and \( f_{\text{ppm}}^1 \) corresponding to the phase models \( \Phi_{\text{rcm}} \), \( \Phi_{\text{ppm}}^0 \), and \( \Phi_{\text{ppm}}^1 \), where \( f = (2\pi)^{-1} \frac{d\Phi}{dt} \).

Figure 7 shows the refractivity profiles inverted from the high-latitude (profile A) and low-latitude (profile B) RO signals shown in Figures 5 and 6 with the use of the model \( \Phi_{\text{ppm}}^1 \). In the lower troposphere, the standard CDAAC inversion process calculates the L1 bending angle as function of impact parameter by Full Spectrum Inversion and performs the ionospheric calibration by extrapolation of L1–L2 bending angles from higher altitudes where L2 is recorded in PLL mode, as described in section 2.3.
Solid and dotted lines correspond to internal and external removal of the NDM. For the high-latitude occultation (profile A) there is no difference between refractivities retrieved with internal and external removal of NDM, while for the low-latitude occultation (profile B) there is a noticeable difference.

4. Adjustment of the Postprocessing Frequency Model

Theoretical estimates show that the frequency model $f_{ppm}$ used in this study, described in detail by Sokolovskiy [2001b], should be accurate to within about 15 Hz. Experimental evaluation of this accuracy is possible with the use of sliding spectrograms of real RO signals.

[35] Figures 8 and 9 show the RO signals $u(t)$ downconverted with the model $f_{ppm}$ in the form of sliding spectrograms for the high-latitude and low-latitude occultations shown in Figures 5 and 6. The sliding spectrograms show in gray scale the normalized amplitudes of the Fourier spectrum calculated in a sliding window of 64 samples (1.28 s) as function of the frequency and the HSL of the center of the window. Figures 8a and 9a and Figures 8b and 9b show the spectrograms after internal and external removal of NDM, respectively. It is seen that internal removal of

Figure 6. Same as Figure 5, but for low-latitude RO signal.
the NDM despreads the spectrum in the regions of low SNR (we note that this artificial despeading due to wrong correction for half-cycle jumps does not recover any real signal). In regions with sufficient SNR there is no difference between internal and external removal of NDM for the high-latitude occultation (Figure 8). For the low-latitude occultation (Figure 9), the external removal of NDM resolved the multipath propagation at \(-50 \text{ km} < \text{HSL} < -30 \text{ km}\), which is otherwise not resolved with the internal removal of NDM. This is consistent with the differences in inverted refractivities in Figure 7. In the spectrograms for the high-latitude occultation (Figure 8) one can see the reflected signal which manifests itself as a weak subsignal appearing at HSL about \(-25 \text{ km}\) and converging with direct signal at HSL about \(-60 \text{ km}\) where both disappear.

Reflected signals in GPS RO, most frequently observed in high latitudes, were described by Beyerle et al. [2002]. For each of the sliding spectrograms shown in Figures 8 and 9, the frequency \(f_c\) corresponding to the local center of the spectrogram at a given HSL is estimated by a method described by Sokolovskiy [2001b]. The \(f_c\) provides an experimental estimate of the error of modeling of the mean RO signal frequency by \(f_{ppm}\). Figures 10a and 10b show \(f_c\) as the function of HSL, determined from the spectrograms in Figures 8b and 9b. The maximum frequency mismodeling is about 4 Hz for the high-latitude and about 10 Hz for the low-latitude occultations.

The frequency mismodeling \(f_c(t)\) determined from the sliding spectrograms can be used to adjust the frequency model: \(f_{ppm} = f_{ppm}^0 + f_c\). Figures 11a and
11b show the sliding spectrograms of the high-latitude and low-latitude RO signals down-converted with the adjusted phase model $\Phi_{\text{ppm}}^1$ and external removal of NDM.

Figures 12a and 12b show the frequency $f_c$, estimated from the spectrograms in Figure 11, which is close to zero. Adjustment of the frequency model by shifting the mean frequency to zero is necessary for not only minimizing the number of half and full cycle slips in the connected phase but, most important, for the elimination of a possible bias when positive and negative phase slips do not cancel.

Another simple and computationally fast approach for adjustment of the frequency model $f_{\text{ppm}}^0$ can be based on the smoothed frequency $f_{\text{sm}}$ of the signal $u$ (after down conversion with the model $\Phi_{\text{ppm}}^0$, removal of the NDM and connection of the phase), i.e., $f_{\text{ppm}}^1 = f_{\text{ppm}}^0 + f_{\text{sm}}$.

Figure 13 shows the raw ($f$) and the smoothed ($f_{\text{sm}}$) frequencies of the down-converted RO signals as functions of HSL, for the high-latitude (Figures 13a and 13b) and low-latitude (Figures 13c–13e) occultations, before and after the adjustment of the frequency model. The low-latitude occultation requires more than one iteration, i.e., $f_{\text{ppm}}^1 = f_{\text{ppm}}^0 + f_{\text{sm}}$. This is due to the fact that some full cycle slips in the phase that exist after down conversion with $\Phi_{\text{ppm}}^0$ are resolved after down conversion with $\Phi_{\text{ppm}}^1$. With the use of the sliding spectrograms (that are based on the complex signal and thus insensitive to full cycle ambiguities) the adjustment of the frequency model does not need iterations. Also, the sliding spectrograms reveal the multipath structure of the RO signals. For these reasons the sliding spectrograms rather than the smoothed Doppler are used for adjustment of the frequency model in the routine processing at the CDAAC.

When the frequency mismodeling with the use of $f_{\text{ppm}}^0$ does not exceed half of the sampling frequency, i.e., 25 Hz, then aliasing does not result in errors of determining $f_c$ and the adjusted model $f_{\text{ppm}}^1$ becomes independent of the $f_{\text{ppm}}^0$. The model $f_{\text{ppm}}^0$ used in this study
satisfies this condition. Figure 14a shows maximum frequency mismodeling \( f_c \) determined from the sliding spectrograms after down conversion of RO signals with the model \( f_{\text{ppm}}^0 \) for all COSMC occultations from 1 September 2006. It is seen that only a few samples are close to ±25 Hz, which indicates that aliasing effects are not significant. A bias can be seen that can be related to the bias in the bending angle (refractivity) climatology and, to a certain extent, to the method of calculation of \( f_{\text{ppm}}^0 \). Figure 14b shows the maximum frequency mismodeling \( f_c \) after down conversion with the model \( f_{\text{ppm}}^1 \). Comparison of Figures 14a and 14b shows that adjustment of the frequency model substantially reduces the spread and eliminates the bias (analysis shows that maximum residual frequency mismodeling corresponds to regions of RO signals with low SNR). Thus the use of feedback from the whole RO signal in postprocessing (different from the recursive feedback in real time in PLL) makes the postprocessing of the OL RO signals independent on the first guess frequency model.

5. Statistical Evaluation of the Differences in Inversion Results Obtained With Different Frequency Models and Different Removal of the NDM

Besides analyzing individual occultations, it is important to statistically estimate the differences in the inverted refractivities related to the use of different frequency models and different methods of removal of the NDM. These differences are expected to be more significant in low latitudes where AIM of the phase is larger. For statistical comparison, COSMIC occultations were used from September 2006 between 30 S and 30 N. The OL RO signals were postprocessed with internal and external removal of NDM and with \( f_{\text{ppm}}^0 \) and \( f_{\text{ppm}}^1 \) phase models. In order to perform statistical comparison on the same data set we use only those occultations for which the externally recorded NDM bit sequences were available (but process them with external and internal removal of NDM). The NDM bit sequences were not available for all occultations, because of temporal operational failures of some ground based NDM-recording receivers.

[42] For interpretation of the differences between the inversion results we note that inversions of RO signals are affected by many different kinds of errors, and none of the retrieved refractivities can be considered the “truth.” However, the use of internal removal of NDM (compared to the external) and the use of the first guess phase model \( f_{\text{ppm}}^0 \) (compared to the adjusted \( f_{\text{ppm}}^1 \)) may only increase, but not reduce the overall inversion error.
because the introduced errors are statistically independent of other errors. Thus, the processing with external removal of NDM and with $\Phi_{\text{ppm}}$ should introduce the smallest errors and may be used as the reference refractivity.

Figure 15 shows the mean fractional differences of inverted refractivities from the reference refractivity (introduced above) as functions of height in the troposphere. It is seen that with the use of external removal of NDM there is almost no difference between the use of the adjusted $\Phi_{\text{ppm}}$ and the first guess $\Phi_{\text{ppm}}$ phase models (solid line). This indicates that the first guess model used in this study, based on CIRA-86aQ climatology, works well for connection of the phase of the OL RO signals with external removal of NDM. Internal removal of NDM results in larger errors with the use of both first guess and adjusted phase models (dotted and dashed lines). The difference between the dotted and dashed lines indicates that adjustment of the phase model results in reduction of the inversion errors below 3 km and some increase of errors between 3 and 5.5 km (the latter needs further exploration). Thus external removal of NDM is more important as it allows substantial reduction of inversion errors (up to 0.5% below 2 km) while adjustment of the Doppler model based on CIRA-86aQ is less important (however, the adjustment may be more important for another first guess model).

External removal of NDM and adjustment of the frequency model for reduction of the inversion errors of the OL RO signals are most important in low latitudes and less important in high latitudes. Nevertheless, CDAAC processes all OL RO signals from COSMIC satellites with the adjustment of the frequency model and with the external removal of NDM (unless externally recorded NDM bit sequences are not available).

6. Conclusions

Postprocessing of the GPS RO signals recorded in OL mode consists of removal of the NDM and connection of the phase between samples. It is important that this process does not introduce errors (biases) in the connected phase. For this purpose the signal is considered as the sequence of complex samples where the frequency model used for down conversion and noise filtering in the receiver does not play any role. The down conversion in the postprocessing is done in two steps: (1) with the first guess frequency model which is based on the postprocessed navigation solution and refractivity climatology; (2) with the adjusted frequency model, by determining the mean residual frequency of the RO signal after down conversion with the first guess model. Since the accuracy of the first guess model based on CIRA-86aQ is better than half of the sampling bandwidth (25 Hz), the adjustment of the frequency model makes the results of the postprocessing independent of the first guess model. The NDM can be removed from OL RO signals internally (similarly to how this is done in PLL), or with use of the externally recorded NDM bit sequence. Statistical comparison for tropical occultations shows that internal removal of the NDM results in substantial increase of refractivity inversion error (bias) up to 0.5% in the lowest 2 km. Thus external recording of the NDM bit sequences for inversions of OL RO signals is important, especially, for climate studies. With external removal of NDM the first guess frequency model based on CIRA-86aQ refractivity climatology is already good enough and its adjustment does not affect the inversion results significantly.

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