The Neutral Atmosphere of Venus as Studied with the Mariner V Radio Occultation Experiments

Gunnar Fjeldbo and Arvydas J. Kliore
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California

AND

Von R. Eshleman
Center for Radar Astronomy, Stanford University, Stanford, California
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The Mariner V radio occultation measurements at 423.3 and 2297 MHz (S band) are used to derive profiles in height of refractivity, molecular number density, pressure, temperature, and dispersive radio-frequency absorption for the atmosphere of Venus. The measurements cover heights between about 90 and 35 km (above a reference surface at a radius of 6050 km), over a pressure range from about $4 \times 10^{-4}$ to 7 atmospheres. Results obtained on the day and night sides are remarkably similar. The 90- to 60-km region contains inversion and thermal layers with the minimum temperature being at least as low as 180 °K. The average temperature lapse rate is 4 °K/km between 80 and 60 km. From 60 to 50 km the lapse rate is about $10^{-5}$°K/km, equal to the dry adiabatic rate for CO2. No radio absorption was observed above 50 km. In the 50- to 35-km height region, the lower-frequency signal was not absorbed, but the S-band signal suffered an approximately constant loss of $4 \times 10^{-3}$ dB per kilometer of propagation path. Assuming that the agent causing the microwave loss has negligible refractivity, there is a minimum in the temperature lapse rate between 50 and 45 km altitude. Below this transition region, the atmosphere may be slightly superadiabatic with the temperature reaching approximately 500 °K at the lowest level of measurement. The temperature and microwave loss profiles suggest the presence of two different cloud systems separated in altitude by about 10 km.

I. INTRODUCTION

TWO independent radio propagation experiments were conducted during the occultation of Mariner V by Venus on 19 October 1967. One of these experiments utilized the carrier frequency of the S-band tracking and communications system. This signal was generated in the spacecraft and received with the NASA Deep Space Net’s 210-ft dish at Goldstone, California (Kliore et al. 1967, 1969). The other experiment was performed by transmitting two harmonically related frequencies, 49.8 and 423.3 MHz, from a 150-ft parabolic dish at Stanford, California, and by receiving the signals with phase-locked receivers in the spacecraft (Mariner Stanford Group 1967; Fjeldbo and Eshleman 1969).

Both of the radio occultation experiments conducted with Mariner V provided valuable data on the physical properties of the neutral and ionized regions in the atmosphere of Venus. The primary purpose of this report is to present new results for the neutral atmosphere from a combined analysis of these two sets of measurements. A special effort is made to bring out the structure of the atmosphere by utilizing a minimum amount of filtering in the data analysis.

Figure 1 shows the two locations where the radio beams probed the atmosphere. Immersion occurred on the night side and emersion on the day side of the planet. The following sections summarize the results deduced from the data.

II. THE NIGHT-SIDE ATMOSPHERE

In this section, we describe how integral inversion of the S-band Doppler data and the 423.3-MHz amplitude data has been employed to calculate two independent refractivity profiles for the night-side atmosphere of Venus. Each refractivity profile has, in turn, been utilized to compute pressure and temperature as a function of altitude. The resulting temperature profiles, in particular, show remarkable similarity both in detailed structure and over-all shape.

After describing the thermal profiles, we proceed to the analysis of the atmospheric microwave loss at 2297 MHz. The amplitude variations observed on the S-band link during the occultation measurements were caused by atmospheric loss and defocusing. In order to calculate the loss profile, the defocusing must be calibrated out. The 2297- and 423.3-MHz signals suffered the same amount of defocusing. We have, therefore, simply utilized the difference between the amplitudes of the two signals to calculate the dispersive loss.

![Fig. 1. Map showing the locations on Venus where the radio beams probed the atmosphere. The zero-meridian plane contains the Earth at inferior conjunction. The regions denoted α and β are pronounced radar scatterers (Goldstein 1965).](image-url)
profile of the atmosphere. This computation yields essentially the S-band loss since no loss was expected at the lower frequency. Finally, we show that approximately the same loss profile can be deduced from the use of only S-band data. In the latter calculation, the defocusing was computed from the Doppler data.

Only the properties of the neutral atmosphere are discussed here. The dual-frequency experiment provided data on the night-time ionosphere, but the on-board crystal oscillator used to generate the S-band signal did not have sufficient frequency stability to allow detection of this plasma. Thus, it is only with regard to the neutral atmosphere that a combined analysis of the night-side data from both experiments offers special advantages.

Reception of the tracking signal from Mariner V was with phase-locked loop (closed loop) and wide band (open loop) receiver channels. The phase-locked loop channel provided amplitude and frequency data in real time. The output from the wide-band receiver channel was recorded in analog form for later digitization and analysis.

Figure 2 shows the atmospheric frequency perturbation observed on the S-band link during immersion into occultation. These data were obtained from digital processing of the wide-band recordings. The signal frequency was sampled at a rate of 5 to 10 per second. The highest rate was used while probing near the top of Venus' atmosphere where the transverse velocity of the ray path was greatest (about 7.2 km/sec). The phase-locked loop receiver channel provided valuable data redundancy, but the sampling rate was less convenient (one frequency sample per second) and severe atmospheric defocusing caused the receiver to lose lock after probing the atmosphere for about 3 min.

We now outline how these observations may be utilized to determine the vertical refractivity profile of the atmosphere. For a more detailed description, the reader is referred to Appendices A through C. Appendix A shows how the observable properties of the signal are related to the angle of refraction and the ray asymptote altitude. Appendices B and C describe techniques that may be utilized to compute the atmospheric refractivity profile from the geometric ray parameters.

Let $K$ denote the number of data points describing the Doppler curve shown in Fig. 2. Each Doppler point corresponds to one position of the radio ray which goes from the spacecraft to the tracking station (see Fig. 20). In what follows, we shall consider each of these ray positions as one ray. Thus, the $K$ Doppler points correspond to $K$ radio rays penetrating the atmosphere down to different depths (see Fig. 22). Knowing the spatial coordinates of the spacecraft, the planet and the tracking station, and assuming a spherically symmetric atmosphere, each Doppler point can be utilized to determine the two geometrical ray parameters: the angle of refraction, and the altitude of the ray path asymptote. These parameters are defined in Fig. 20. The changes in the angle of refraction with depth

![Graph](image-url)

**Fig. 2.** Atmospheric Doppler perturbation observed on the S-band link during immersion: The data were obtained from a wide-band, fixed-tuned receiver channel at Station 14. At 3 min past encounter (periapsis), the beam was bent approximately 17.6 deg.

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creasing altitude can, in turn, be used to calculate how the atmospheric refractivity varies with height. In short, this computation is performed by dividing the atmosphere into $K$ spherical layers. Starting with the top ray that passes tangentially through the upper layer, one computes the refractivity which this layer must have in order to yield the appropriate ray bending. The second ray from the top is passing through two atmospheric layers. The bending which the top layer imposes on the second ray can readily be subtracted. The remaining residual of the angle of refraction is used to determine the refractivity of the second layer. Repeating these computations for the remaining rays gives the entire refractivity profile.

Figure 3 shows the refractivity profile deduced from the S-band Doppler measurements made during immersion into occultation. These refractivity data cover the 90- to 35-km altitude region. Above 90 km, there was not enough neutral gas to be detected. As the lowest point on the radio ray approached the 35-km altitude level, the radius of curvature of the ray path became comparable to the distance to the planet's center of mass and the signal was extinguished by severe defocusing. We are here up against a fundamental limitation of radio occultation experiments, namely that no propagation path can probe tangentially below the altitude level where the radius of curvature of a horizontal ray is equal to or smaller than the distance to the center of mass of the planet. Calculations based on downward extrapolation of the refractivity profile shown in Fig. 3 indicate that the radio beam reached down to within a fraction of a kilometer of this superrefractive portion of the atmosphere.

For the purpose of providing altitude scales for the figures of this report, we have assumed a radius of 6050 km at the surface of Venus. Computations based on radar data (Ash et al. 1968; Melbourne et al. 1968) and on both radar data and Mariner V tracking data (Anderson et al. 1968; Eshleman et al. 1968) have yielded values for the radius ranging from 6048 to 6056 km.

Direct composition measurements performed with the USSR landers Venera 4 through 6 indicate that carbon dioxide constitutes 93% to 97% of the atmosphere of Venus (Vinogradov et al. 1968; Avduevsky et al. 1970; Kuzmin 1970). Nitrogen and inert gases apparently make up 2% to 5% by volume. Using this information together with experimental data relating refractivity to gas density (Essen and Froome 1951; Tyler and Howard 1969), one can convert the refractivity profile of Fig. 3 into number density as indicated by the upper scales in this figure. The number density profile can, in turn, be utilized to determine the temperature and pressure profiles of the atmosphere (Fjeldbo and Eshleman 1968). This computation is performed by utilizing the equation for hydrostatic equilibrium and
integrating the density from the top of the detectable atmosphere and downward. The temperature at the upper boundary enters into the solution as a constant of integration. Figures 4 and 5 show the resulting temperature and pressure profiles obtained by using two different boundary temperatures near 90 km altitude. Between 80 and 90 km altitude, one can only use the measurements to determine vertical variations in temperature relative to the value selected at the top of the atmosphere. However, below about 80 km altitude the two profiles of Fig. 4 are seen to merge and the result is therefore essentially independent of the choice of boundary condition.

The computations illustrated in Figs. 3 through 5 were carried out for the case of 100% CO₂ and for 95% CO₂ and 5% N₂ (by volume). The remaining uncertainty in the N₂ abundance is seen to have only a minor effect on the results. The boundary temperature was set equal to 150 and 250 °K for 100% CO₂. These values are thought to represent extremes at 90 km altitude.

In addition to the uncertainties described above, the computed profiles are also affected by instabilities in the radio system. (The spacecraft oscillator that generated the S-band signal was only stable to a few parts in 10⁴.) An error analysis based on Doppler data taken prior to occultation shows that this source of error influences the computed profiles most near the top of the atmosphere and it can cause effects comparable to the thermal layers that appear above 80 km in Fig. 4. Below 70 km altitude, oscillator instabilities contributed errors less than ±1% to the derived pressure and temperature profiles.

In view of the experimental uncertainties described above, it was judged important to derive an entirely independent set of temperature and pressure profiles (see Figs. 6 and 7) from data obtained with the dual-frequency experiment. In the latter experiment, 49.8- and 423.3-MHz signals were transmitted to the spacecraft where the amplitudes and the differential dispersive Doppler were measured every 0.6 sec. The 49.8-MHz signal amplitude was affected quite severely by ionospheric scintillations and multipath propagation, and unambiguous inversion of these data is therefore not possible. However, the 423.3-MHz amplitude data illustrated in Fig. 8, are well suited for integral inversion and were utilized to calculate the temperature and pressure profiles shown in Figs. 6 and 7.

In order to invert the 423.3-MHz amplitude data, it was necessary to make the following assumptions:

(i) All observed changes in the received power were caused by atmospheric defocusing. (None of the gases detected with the USSR landers would produce a measurable absorption at this radio frequency.)
(ii) No multipath propagation occurred, i.e., the ray asymptote altitude was decreasing monotonically with time during immersion.

The amplitude of the probing signal is very sensitive to the structure of the atmospheric temperature profile. For instance, when the 423.3-MHz amplitude data shown in Fig. 8 were inverted, the two dips in the signal level observed at 2.25 and 2.0 min before encounter yielded minima in the temperature lapse rate near 48.5 and 46.5 km altitude, respectively. The thickness of these low-lapse-rate layers is of the order of 1 km. The same minima were also detected with the S-band Doppler measurements. These temperature changes are so small that they can barely be seen in the plotted profiles, yet the amplitudes of the signals were strongly affected. [It is also possible that local refractivity irregularities at the 48.5- and 46.5-km altitude levels may have caused the observed signal perturbations. Spherically shaped blobs with radii of about 1 km and temperatures of the order of 10 °K above ambient have, for example, been invoked as a possible interpretation (Fjeldbo and Eshleman 1969).]

Inspection of Figs. 4 and 6 reveals that the temperature profiles obtained from the two experiments are remarkably similar at all levels. A feature of interest is the temperature transition in the height range of 45–50 km, between two regions of larger lapse rate. This feature, which also appears in the day-side profile, has not been recognized in previous analyses and discussions of Mariner V and Venera 4, 5, and 6 data, although in retrospect it appears that the few data points of Venera 6 near this height may show a similar transition. Also note, for example, the fine structure in the temperature profiles of Figs. 4 and 6, in the 60–90-km altitude region. Since similar temperature variations were detected with both radio links, they could not have been produced by equipment anomalies such as variations in the S-band frequency derived from the spacecraft’s local oscillator, or by changes in the 423.3-MHz system gain. Scintillations in the terrestrial atmosphere can also be ruled out since the two radio signals traversed this medium at different locations and at times separated by approximately 9 min. Thus, we conclude that the 60–90-km altitude region on Venus, which in density is equivalent to the terrestrial stratosphere, has a complex temperature profile including several relatively warm and cold layers.

Figure 8 shows a comparison of the 423.3- and 2297-MHz amplitude changes produced by the atmosphere of Venus. Both radio links utilized circularly polarized waves so there were no variations in the
Fig. 7. Pressure profiles determined from the 423.3-MHz amplitude data.

Fig. 8. Comparison of the amplitude variations produced by the atmosphere of Venus at 423.3 and 2297 MHz during immersion: No filtering was employed. The 423.3-MHz spacecraft receiver went out of lock 0.3 min before encounter.

Fig. 9. Atmospheric propagation loss at S band versus the altitude of the lowest point on the radio ray: For the full-drawn curve, the defocusing was removed by using the 423.3-MHz amplitude data. For the stippled curve, the defocusing was computed from the Doppler data.
signal levels due to ionospheric Faraday rotation. The tracking signal was transmitted from a high-gain spacecraft antenna. Refraction in the atmosphere of Venus changed the direction of propagation and therefore also the antenna gain during the occultation measurements. This effect has been removed from the data shown in Fig. 8.

The defocusing produced by differential refraction in Venus’ atmosphere is essentially the same at both wavelengths. The difference between the two curves in Fig. 8 is therefore a measure of dispersive propagation loss, either absorption or scattering. The full-drawn curve in Fig. 9 shows this differential loss as a function of the altitude of the lowest point on the radio path. As indicated by the stippled curve in the same figure, the S-band loss can also be determined by computing the atmospheric defocusing from the Doppler data. Limitations of this approach are discussed below.

In the process of deducing the refractivity profile from the S-band Doppler data, we computed the changes in the angle of refraction (α) and the ray asymptote miss distance (δ) as explained in Appendix A. Using α(δ) and the spatial coordinates of the transmitter, the planet, and the receiver, one can compute the changes in the defocusing with time. The problem is to make this procedure work also with regard to scintillations caused by relatively minor structure in the atmosphere. The magnitude of the frequency perturbation produced by an irregularity in the refractivity is proportional to the radial velocity of the radio beam, while the corresponding defocusing is independent of this velocity. Computing the defocusing from the Doppler measurements works quite well at high altitudes where the beam moved rapidly down through the atmosphere. However, near the level of critical refraction, the radio ray moved so slowly downward that the Doppler scintillations were masked by noise. It is, therefore, difficult to use this approach to accurately calibrate the defocusing from the amplitude data at the lower levels of measurement.

Figures 10 and 11 show the vertical changes in the loss coefficient in decibels per kilometer of ray path length. These profiles were obtained by inverting the data show in Fig. 9. Since the detailed variations in the two loss profiles are not the same even after some smoothing, we are unable to infer structure other than the change from zero above about 50 km, to an approximately constant value of 4 to 5×10−8 dB/km of path length for S band at altitudes between 37 and 50 km.

The calculations presented in Figs. 10 and 11 represent a refinement of first-order results published earlier (Kliore et al. 1969). In the initial analysis of the microwave loss, the defocusing was computed from a smooth model of the refractivity distribution and no attempt was made to calibrate out amplitude scintillations produced by refractivity irregularities in the atmosphere of Venus. As a result, the earlier calculations gave too high a microwave loss at altitude levels where the radio beam was affected by scintillations.

III. THE DAY-SIDE ATMOSPHERE

Figures 12 through 14 illustrate the three types of data employed in the analysis of the sunlit side of the atmosphere. The closed-loop receiver channel at Station 14 reacquired the signal approximately 20.8 min past encounter. The radio ray had then reached an altitude of 43 km. Prior to this time, the open-loop receiver channel was the only source of data. The 423.3-MHz spacecraft receiver reacquired lock 21.1 min past encounter.

As in Sec. II, we also restrict our discussion to the atmosphere below 90 km here. Ionospheric data from
the two occultation experiments have been used previously to define both the main day-side layer and the higher “plasmapause” (Kliore et al. 1967; Mariner Stanford Group 1967; Fjeldbo and Eshleman 1969). The 49.8-MHz signal was severely affected by critical refraction on the bottom side of the ionosphere and

Fig. 13. Unfiltered amplitude perturbations produced by the atmosphere of Venus at 2297 MHz during emersion: Changes in the spacecraft antenna gain, caused by refraction, have been removed from the data.

Fig. 14. Atmospheric Doppler perturbation observed on the S-band link during emersion from occultation: Measurements were made with open- and closed-loop receivers at Station 14.

data from this receiver channel were not employed in the analysis described here.

By following the procedures outlined in Sec. II, we have computed the atmospheric refractivity, molecular

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Fig. 15. Refractivity and number density profile deduced from the Doppler data: Pre-inversion filtering was performed by averaging $n(\phi)$, the angle of refraction expressed as a function of the altitude of the ray asymptote, over 0.1-km altitude intervals.
Number density, and temperature and pressure profiles from the S-band Doppler data obtained during emersion. These profiles are shown in Figs. 15 through 17. Similar results have been deduced from the amplitude data in the altitude region where absorption is negligible.

Refraction in the day-side ionosphere of Venus caused the 423.3-MHz radio waves to reach the spacecraft simultaneously via several different paths. The receiver could, of course, only remain phase locked to one signal at a time. It generally locked on to the strongest signal available. Unambiguous inversion of the data is not possible under such circumstances. It was therefore necessary to take a different approach, in order to compare the 423.3- and 2297-MHz data. We chose to utilize the refractivity profile derived from the S-band Doppler data to compute the defocusing expected at the lower frequency. The calculated and observed amplitude changes are compared in Fig. 12. It should be noted that the higher frequency is less sensitive to ionospheric structure than the 423.3-MHz measurements. Also, pre-inversion filtering was employed before the refractivity profile was computed. In spite of these differences, the two curves in Fig. 12 agree quite well. The temperature and pressure profiles derived from the S-band data are, therefore, consistent with the measurements made at the lower frequency.

A comparison of the immersion and emersion profiles reveals that the two sides of the planet are strikingly similar.* For instance, both sides show the transition region at 45–50 km altitude which was described previously, a relatively sudden change in the average lapse rate for regions above and below about 60 km, and inversions and thermal layers in the 60–90-km height range. The average lapse rates above and below 60 km are approximately 4 and 9°C/km, respectively. The last value is in agreement with the Venera measurements which covered the 300 to 600°C temperature regime (Avduevsky et al. 1970). However, between 50 and 58 km (360 to 280°C) we find a lapse rate of about 10°C/km, essentially the dry adiabatic value for this region (Staley 1970). Near the 35-km level, the temperature lapse rate appears to be slightly higher than the adiabatic rate for a dry CO₂ atmosphere, but this is the most difficult region for accurate radio measurement.

* A preliminary study performed by fitting a simple atmospheric model to the 423.3-MHz amplitude data indicated that the day side might be colder than the night side at all altitudes. We are here referring to Figs. 13 and 15 in a paper by Fjeldbo and Eshleman (1969). The atmospheric model employed in that study consisted of two altitude regions, each having a constant temperature lapse rate. Looking back at this earlier work and comparing it closely with the results presented here reveals that the earlier model was too simple for an accurate description of the atmosphere of Venus. In particular, the minimum in the lapse rate which occurs in the 45–50-km region caused defocusing which the model could not reproduce. This effect together with the fact that fewer data were available on the emission side, created the incorrect impression that the day side might have been generally colder than the night side.
Downward extrapolation of the pressure and temperature (at 9°K/km) to a radius of 6050 km yields surface values of approximately 90 atmospheres and 800°K, respectively, although if the temperature lapse rate changes according to the changing dry adiabat, the temperature would be nearer 775°K. A few kilometers difference in the radius of the surface would of course have important effects on these numbers.

The loss suffered by the S-band link during emersion is shown in Fig. 18. The defocusing was removed by utilizing the Doppler data in the manner described in Sec. II. Integral inversion yields the loss coefficient as a function of altitude. The results are shown in Fig. 19.

The 423.3-MHz spacecraft receiver reacquired lock as the radio ray reached an altitude of approximately 45 km. Above this level we have also used the amplitude measurements at the lower frequency to calibrate S-band defocusing. This method gives loss profiles similar to those illustrated in Figs. 18 and 19.

In comparing the day and night sides, we note that the propagation loss appears to cease abruptly near the 50-km level. Below this altitude, the signal loss coefficient is about $4 \times 10^{-3}$ dB/km of path length. To within the accuracy of the amplitude measurements, the loss coefficient may be the same on the two sides of the planet.

The temperature calculations presented in Figs. 4, 6, and 16 were based on the assumption that the atmosphere is completely mixed throughout the altitude region under study. However, microwave absorption by carbon dioxide and nitrogen is negligible near 50 km. Thus, the observed loss profiles suggest that the composition may vary with altitude.

In case an absorbing agent contributes significantly to the refractivity below 50 km, the derived temperature profiles would be inaccurate in this region. Of course, if one knew how the observed loss and refractivity were related to the temperature and number density of the absorbing constituent, one could utilize these two equations together with the equation for hydrostatic equilibrium, to calculate the atmospheric temperature profile and the number density profiles for the absorbing agent and carbon dioxide. However, in the absence of the necessary information concerning the loss mechanism, we shall restrict our discussion to two simple examples which illustrate how an altitude-dependent composition would affect the analysis of the refractivity data.

In what follows, we will assume a temperature profile and utilize the refractivity data to determine constraints on the composition of the atmosphere. Of special interest is the case where:
(i) the atmosphere consists of CO$_2$ and a microwave absorber. The latter constituent is only present below 50 km.
(ii) the temperature distribution is adiabatic below 50 km altitude;
(iii) the atmosphere is in hydrostatic equilibrium.

The two examples given below are based on these three assumptions.

First, we consider the case where for equal number densities, the refractivity of the absorbing agent is approximately the same as that of carbon dioxide. By utilizing the refractivity data to calculate the mean molecular mass as a function of altitude, one arrives at a profile which has a maximum between 45 and 50 km. The peak value is about 10% higher than the mass of the CO$_2$ molecule. It is interesting to note that a similar change was deduced from the Venera 4 measurements (Avduevsky et al. 1968).

Finally, we consider the situation where the microwave absorber does not significantly alter the mean molecular mass of the atmosphere. This assumption puts a constraint on the refractivity distribution of the absorbing agent. Our analysis shows that the absorber, in this case, must contribute at least 57 and 130 to the atmospheric refractivity at 45 and 40 km altitude, respectively. The structure of the atmosphere is discussed further in the next section.

IV. PRELIMINARY INTERPRETATIONS

Our report concerns itself primarily with the new results derived from the radio occultation measure-

![Fig. 18. Atmospheric propagation loss at S band versus altitude of the lowest point on the radio ray.](image_url)

ments. However, we also wish to comment on possible interpretations of several of the detailed features in the profiles, and to compare our results with measurements and model studies conducted by other investigators.

Because of the high surface temperature of Venus, a number of vapors may exist in the atmosphere and condensation of such constituents could play an important role in the thermal budget at various altitude levels. For example, the changes in temperature lapse rate near 60 and 45 km altitude, and the sudden appearance of S-band loss below 50 km, are suggestive of effects of condensible vapors. In order to illustrate this type of phenomenon, we discuss some of the possible effects of water. It is generally accepted that water is present in the atmosphere of Venus. However, the amount is a subject of contention [see for example Rea and O’Leary (1968); Sagan and Pollack (1969); Gierasch and Goody (1970)].

The Venera spacecraft indicated water-vapor mixing ratios of between $10^{-4}$ to $10^{-8}$ (by mass) in regions below the visible clouds (Avduevsky et al., 1970; Vinogradov et al. 1968). With the new temperature and pressure profiles, we illustrate in Fig. 16 that condensation of water clouds would occur at an altitude of 58 km for a mixing ratio of $10^{-4}$, assuming that there were suitable condensation nuclei. For a mixing ratio of $10^{-8}$, ice clouds would sublime at 68 km. At the height where condensation or sublimation starts, a sudden reduction of lapse rate with increasing altitude is expected, as illustrated by the wet adiabatic curve (for a mixing ratio of $10^{-2}$) in Fig. 16. The amount of the change in lapse rate is less for smaller mixing ratios. Thus, the relatively sudden change in lapse rate which occurs near 60 km altitude might be interpreted as indirect evidence supporting the Venera water measurements.
Alternatively, this lapse rate transition may mark the top of a convective region having the adiabatic lapse rate, and the start of a stable higher region in radiative equilibrium.

Arguments against this much water vapor and against the clouds consisting of water or ice are based primarily on spectroscopic observations. Water-vapor absorption lines have been interpreted as being formed at a temperature of about 240°K, and at a mixing ratio of at most $10^{-4}$, with values of $10^{-6}$ being quoted more often [see, for example, the review of Hunt and Goody (1969)]. At this temperature, no water or ice condensation would occur for such low mixing ratios. Also, some of the initial temperature profiles derived from the S-band occultation data showed an isothermal region at about 250°K above an altitude of 60 km, and no water condensation would occur under these conditions unless the mixing ratio were well above $10^{-4}$.

The new temperature profiles reported here are very important to the question of water vapor. They show that the temperature goes low enough for ice crystals and water vapor to be in equilibrium at mixing ratios certainly as low as $10^{-4}$, and perhaps as low as $10^{-6}$ (see Fig. 16). If the spectroscopic lines could have been formed in ice clouds at temperatures near 200°K instead of at 240°K, the various results on water could be reconciled and the Venus atmosphere would have the higher water content suggested by the Venera measurements. In this regard, it is our understanding that the theory on line formation requires the assumption of a fairly homogeneous scattering region. But the observed fine structure in the temperature profiles in the 60–80-km altitude range seems to imply rather non-homogeneous conditions. This is particularly striking when it is realized that the occultation results are averages over a horizontal extent of several hundred kilometers for structures that is not spherically stratified. While the temperature profiles indicate that on the average the atmosphere is absolutely stable in most of this height range, it would seem that there could be a mixture of cumulus towers in otherwise clear air which is in radiative equilibrium, as suggested by Johnson (1969). The measurements would show characteristics of a mixture of these two regions. Ice crystals carried up in these towers would be expected to spread out at the inversion layer near 80 km altitude. While it is possible that the break in the temperature curve at about 60 km altitude marks the tropopause (i.e., the maximum altitude to which convection currents of the troposphere can reach), the view presented above may make it more appropriate to say that the stratosphere starts at a greater altitude.

The temperature transition region from 50 to 45 km altitude seems to be connected with the transition between zero and a constant S-band loss coefficient. The near constancy of the loss coefficient (from about 50 to 37 km altitude) is not easily explained. Barrett and Staelin (1964) and Ho, Kaufman, and Thaddeus (1966) have published formulas for the absorption of radio waves in CO$_2$ and water vapor. Utilizing their results, we find that pressure-induced absorption by CO$_2$ did not contribute measurably to the observed loss at 50 km, and it would account for only about $10^{-4}$ dB/km near 37 km altitude. For small amounts of H$_2$O in the Venus atmosphere, water vapor (by mass) is expected to be about 200 times more effective as a microwave absorber than is CO$_2$. [It should be noted that the measurements on which this statement is based involved N$_2$–H$_2$O mixtures (Ho, Kaufman, and Thaddeus 1966).] Thus for a mixing ratio of $5 \times 10^{-4}$, for example, the absorption due to CO$_2$ and H$_2$O are expected to be about equal. For a constant mixing ratio, the loss coefficient due to both species would increase in the same way with decreasing altitude, being proportional to the square of the pressure and approximately to the inverse fifth power of the temperature. (Such pressure-induced absorption is also proportional to the square of the radio frequency so that no measurable absorption at 423.3 MHz was expected due to this cause.)

For the case that the atmosphere is assumed to consist only of CO$_2$ and H$_2$O, the refractivity and S-band loss data have been utilized in an attempt to calculate the temperature profile and the number density profiles for H$_2$O and CO$_2$. (The third equation needed in this computation was obtained by assuming hydrostatic equilibrium.) Our analysis of this problem shows that it is not possible to explain the observed loss profile in terms of H$_2$O–CO$_2$ absorption alone. Of course, this conclusion does not preclude water clouds near 60 km altitude.

There are, of course, other possible microwave loss mechanisms. Clouds of dust could cause both absorption and scattering, but the particles would have to be distributed fairly evenly throughout the altitude regime under study. Dust has previously been suggested as a possible explanation for the microwave spectrum of Venus (Barrett and Staelin 1964) and for the Earth-based radar results which show a decreasing cross section towards shorter wavelengths (Evans and Hagfors 1966; Ingalls and Evans 1969). The amount of dust required to produce the observed loss is too small to significantly alter the temperature profiles derived in Secs. II and III. Another possible cloud type which might provide the necessary path loss consists of particulate mercury halides (Lewis 1969; Rasool 1970), but they also would need to be more uniformly distributed than has been suggested previously. However, related vapors may be involved in the absorption with the change in the calculated temperature lapse rate being due to condensation of the clouds, or to the refractivity signature of the vapors. For a more general discussion of other possible constituents, the reader is referred to Rea and O'Leary (1968) and Gierasch and Goody (1970).

The temperature lapse-rate transition just below 50
km is also suggestive of a stable zone between two turbulent convective layers. Convection in the lower zone is of course needed to support clouds that might be causing the radio loss. We also note that the calculated lapse rate below about 40 km altitude appears slightly superadiabatic. These characteristics appear to lend support to the models of the atmosphere of Venus which are based on an internal planetary source of heat, with dust or condensate clouds being supported by strong convection (Hansen and Matsushita 1967; Gierasch and Goody 1970). The deposition of solar energy might then be primarily at the top of the clouds.

There is a general point that should be made about whatever is causing the measured S-band loss. We have shown that it is not due to CO$_2$ in the height range of this measurement. However, Muhleman (1970) has concluded from the radar and radio emission experiments that CO$_2$ could account for all of the total optical depth of the atmosphere for microwaves, if the surface pressure were as high as 80 atmospheres. Since nearly all experimenters now place the surface pressure this high or higher, we are hard pressed to allow an additional loss mechanism. But our results show that there must be such an additional cause of microwave loss. We found that the amount of this loss increases with increasing frequency, and it may have the same frequency-squared dependence as does the CO$_2$ absorption. Even if the dependence is as weak as the first power of frequency, we conclude, since the total added effect must be small, that the cause of the added (non CO$_2$) loss must be either localized in altitude, or at least not increasing appreciably with decreasing altitude, below the region measured in this experiment. This characteristic seems to imply rather strongly that a condensable material is involved.

While the new profiles of atmospheric parameters for Venus will probably not make it possible to define a unique model, this more detailed information should narrow the range of possibilities. It is planned that dual-frequency occultation measurements of improved accuracy will be conducted during the 1973 Mariner Venus–Mercury mission. Of particular importance to the study of absorption and the nature of the clouds is the fact that the new measurements will include a higher frequency than was used in the experiments reported here, so that it may be possible to measure the frequency dependence of the loss as well as to improve the accuracy of the loss profile.

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APPENDIX A:

RAY PATH PARAMETERS

During the occultation measurements, refraction in the atmosphere of Venus perturbed the frequency and amplitude of the received signals. The purpose of this appendix is to describe briefly how the observed perturbations are related to the two geometrical ray parameters: the angle of refraction ($\alpha$), and the distance ($a$) from the planetary center of mass to the ray path asymptote.

Figure 20 illustrates the geometry of the two occultation experiments. In the case of the S-band experiment, the signal traveled along a ray going from the spacecraft to the tracking station. The signal transit time was about 4 min and 26 sec. At any one instant in time, the cylindrical coordinate system shown in Fig. 20 was chosen so that the $z$ axis passed through the position which the tracking station would occupy 4 min and 26 sec later. The origin of this coordinate system was chosen at the center of mass of Venus.

Based on classical physics, the atmospheric frequency perturbation ($\Delta f$) is given by

$$\Delta f = \left( f_s - f_e \right) \left( \frac{v_{ts}}{c} \sin (\beta_s - \beta_e) - \frac{v_{ts}}{c} \sin (\beta_s + \beta_e) \right)$$

$$+ f_s \left( \frac{v_{ts}}{c} \sin (\delta_s - \delta_e) + \frac{v_{ts}}{c} \sin (\delta_s + \delta_e) \right)$$

$$- \left( f_s - f_e \right) \left( \frac{v_{ts}}{c} \sin (\beta_s - \beta_e) - \frac{v_{ts}}{c} \sin (\beta_s + \beta_e) \right)$$

$$+ f_s \left( \frac{v_{ts}}{c} \sin (\delta_s - \delta_e) + \frac{v_{ts}}{c} \sin (\delta_s + \delta_e) \right),$$

where

- $f_s$ = radio frequency radiated from the spacecraft,
- $c$ = velocity of light in vacuum,
- $v_{ts}$ = spacecraft velocity in the radial direction,
- $v_{ns}$ = spacecraft velocity in the $\delta$ direction,
- $v_{nt}$ = velocity of the tracking station in the radial direction,
- $v_{nt}$ = velocity of the tracking station in the $\delta$ direction.

The angles involved in the above equation are defined in Fig. 20. Velocity components perpendicular to the $rz$ plane do not affect the Doppler shift.

Assuming that the refractivity distribution of the atmosphere is spherically symmetric, both ray path
Fig. 20. Occultation geometry: The illustrated ray path is bent in the ionosphere where the refractive index is less than 1.

The ray asymptotes will have the same distance (α) from the planet's center of mass. This assumption gives a second equation,

\[-z_t \sin (\delta_t - \delta_i) = (r_i^2 + z_i^2)^{1/2} \sin (\beta_e - \gamma - \beta_t). \tag{A2}\]

When the coordinates of the spacecraft and the tracking station are known, one can utilize equations (A1) and (A2) to simultaneously solve for the two ray-path angles \(\delta_t\) and \(\beta_t\). In order to perform this computation, it is convenient to replace \(\delta_t\) and \(\beta_t\) by \(\delta_t + \Delta\delta_t\) and \(\beta_t + \Delta\beta_t\), respectively, and linearize the transcendental equations with regard to \(\Delta\delta_t\) and \(\Delta\beta_t\). This rearrangement yields

\[b_{11}\Delta\delta_t + b_{12}\Delta\beta_t = k_1, \tag{A3}\]
\[b_{21}\Delta\beta_t + b_{22}\Delta\delta_t = k_2, \tag{A4}\]

where

\[b_{11} = -v_{st} \sin (\beta_e - \beta_t) + v_{st} \cos (\beta_e - \beta_t),\]
\[b_{12} = -v_{st} \cos (\delta_e - \delta_t) + v_{st} \sin (\delta_e - \delta_t),\]
\[b_{21} = (r_i^2 + z_i^2)^{1/2} \cos (\beta_e - \gamma - \beta_t),\]
\[b_{22} = z_t \cos (\delta_e - \delta_t),\]
\[k_1 = \frac{\Delta f}{f_s} + v_{st}[\cos (\beta_e - \beta_t) - \cos \beta_e] + v_{st}[\sin (\beta_e - \beta_t) - \sin \beta_e] - v_{st}[\sin (\delta_e - \delta_t) - \sin \delta_e] - v_{st}[\cos (\delta_e - \delta_t) - \cos \delta_e],\]
\[k_2 = z_t \sin (\delta_e - \delta_t) + (r_i^2 + z_i^2)^{1/2} \sin (\beta_e - \gamma - \beta_t).\]

The new set of equations may be utilized to determine \(\delta_t\) and \(\beta_t\) as a function of time. A convenient approach to this problem is to start with rays passing outside the atmosphere where both \(\delta_t\) and \(\beta_t\) are zero and then proceed to rays at lower altitudes. For each ray, one makes an initial estimate of \(\delta_t\) and \(\beta_t\) based on rays at greater altitudes and determines the corrections \(\Delta\delta_t\) and \(\Delta\beta_t\) from Eqs. (A3) and (A4). The new values for \(\delta_t\) and \(\beta_t\) are fed back into the linearized equations and the next set of corrections are computed. For each ray,

Fig. 21. Angle of refraction versus the altitude of the ray asymptote: This profile was computed from the 423.3-MHz amplitude data obtained during immersion.
this iterative procedure is continued until the desired precision is obtained.

Finally, the angle of refraction (α), and the distance (a) from the planetary center of mass to the ray-path asymptote can be computed utilizing the following formulas:

\[ \alpha = \delta_1 + \beta_1 \]  \hspace{1cm} (A5)

\[ a = (r^2 + z^2)^{1/2} \sin (\beta_2 - \beta_1) \]  \hspace{1cm} (A6)

A digital computer was utilized in transforming the atmospheric Doppler perturbations into an α(a) profile. In the programmed solution to this problem, we also made corrections based on special theory of relativity. This calculation was done by perturbing the coefficients of Eqs. (A3) and (A4), and by repeating the iterative procedure described above.

The method delineated here applies to the study of the Doppler data. A similar procedure has been developed for deducing the ray parameters a and α from the 423.3-MHz amplitude data. The latter technique assumes that atmospheric absorption is negligible and that none of the rays crossed each other before reaching the spacecraft receiver. Starting with rays that graze the top of the atmosphere, the amplitude measurements are used point by point to solve for the differential change in the angle of refraction (Δα) with decreasing ray asymptote altitude. Integrating Δα yields α as a function of a. Figure 21 shows the α(a) profile determined in this manner from the 423.3-MHz amplitude data obtained during immersion into occultation.

In Appendices A and B, we shall see how the function α(a) may be utilized to determine the refractive index profile of the atmosphere.

APPENDIX B:
RAYTRACING INVERSION

In this appendix, we discuss how ray-tracing techniques may be utilized to compute the refractive index profile of the atmosphere. It is assumed that the function α(a) consists of K points which have been determined from the amplitude or Doppler data in the manner described in Appendix A.

Figure 22 shows a ray that is being refracted in an atmosphere consisting of K spherical layers. In order to keep mathematical complexity to a minimum, we consider the simple case where each layer has constant refractivity and constant thickness. Tangentially through the middle of each layer passes one ray. Only the mth ray is shown in the figure.

Let \( r_m \) and \( \mu_m \) denote the radius and index of refraction, respectively, for the mth layer. By definition, we require that the radius of closest approach (\( r_{0m} \)) for the mth ray be related to \( r_m \) and \( r_{m+1} \) by

\[ r_{0m} = \frac{1}{2} (r_m + r_{m+1}) \]  \hspace{1cm} (B1)

In order to trace the rays, we need to determine the angle of incidence (\( i_{m,n} \)) and the angle of refraction (\( j_{m,n} \)) at each boundary. Here, the first and the second subscript refer to the ray number and the boundary number, respectively. Employing the law of sines and Snell's law at the first boundary gives the following equations for the mth ray:

\[ \alpha_m = r_1 \sin i_{1,1} \]  \hspace{1cm} (B2)

\[ \sin i_{m,1} = \mu_1 \sin j_{m,1} \]  \hspace{1cm} (B3)

\[ \psi_{m,1} = \frac{1}{2} \alpha_m + i_{m,1} - j_{m,1} \]  \hspace{1cm} (B4)

where

\[ \alpha_m = \text{ray asymptote miss distance for the mth ray,} \]
\[ \alpha_m = \text{total angle which the mth ray is being bent by the atmosphere,} \]
\[ \psi_{m,1} = \text{angle which the mth ray must be bent before it is traced from the first layer to the closest approach point. (ψ is also defined in Fig. 23.)} \]

Equations (B2)–(B4) assume that \( \mu \) is one above the first layer.

Generalizing to the nth boundary gives

\[ r_{n-1} \sin j_{m,n-1} = r_m \sin i_{m,n} \]  \hspace{1cm} (B5)

\[ \mu_{n-1} \sin i_{m,n} = \mu_m \sin j_{m,n} \]  \hspace{1cm} (B6)

\[ \psi_{m,n} = \psi_{m,n-1} + i_{m,n} - j_{m,n} \]  \hspace{1cm} (B7)

For n equal to m, the angle \( \psi_{m,n} \) is zero and the ray equations take the form of

\[ r_{m-1} \sin j_{m,m-1} = r_m \sin i_{m,m} \]  \hspace{1cm} (B8)

\[ \mu_{m-1} \sin i_{m,m} = \mu_m \sin j_{m,m} \]  \hspace{1cm} (B9)

\[ 0 = \psi_{m,m-1} + i_{m,m} - j_{m,m} \]  \hspace{1cm} (B10)

\[ r_{0m} = r_m \sin j_{m,m} \]  \hspace{1cm} (B11)

In an atmosphere with known refractive index profile, the above equations may be used to trace a ray from the first boundary and down to the closest approach point. Because of symmetry, it would not be necessary to perform additional calculations when tracing the ray out again on the other side of the atmosphere.

We now describe how \( \alpha(a) \) and the ray equations may be used to determine \( \mu \) and \( r \) for each layer. For rays passing above the atmosphere, the bending angle is zero and the radius of closest approach is equal to the asymptote miss distance. Let the zeroth layer indicated by the stippled boundary in Fig. 22 represent such a region. For this layer, one can therefore set

\[ \mu_0 = 1 \]  \hspace{1cm} (B12)

and

\[ r_0 = \frac{1}{2} (a_0 + a_{-1}) \]  \hspace{1cm} (B13)

where \( a_0 \) and \( a_{-1} \) denote the asymptote miss distances for the two rays that pass immediately above the first atmospheric layer.
Fig. 22. Refraction of ray path by concentric spherical layers: The illustration is for an increasing refractive index with height.

Employing Eq. (B1), one can now determine the radius of the first layer:

\[ r_1 = 2a_0 - r_0. \]  \hspace{1cm} (B14)

Setting \( m \) equal to 1 and \( \psi_{1,0} \) equal to \( (\alpha_1/2) \) in Eqs. (B8)–(B10), allows us to calculate the refractive index for the first layer:

\[ \mu_1 = a_1(r_1 \sin [(\alpha_1/2) + \sin^{-1}(a_1/r_1)])^{-1}. \]  \hspace{1cm} (B15)

The radius of closest approach for the first ray (\( r_{01} \)) is given by Eq. (B11). Applying Eq. (B1) with \( m \) equal to 1 allows the radius of the second layer (\( r_2 \))
to be determined. Continuing these computations for
the remaining rays yields the entire refractive index
profile of the atmosphere. We note, for instance, that
the radius \(r_m\) and the refractive index \(\mu_m\) of the
\(m\)th layer are determined from the \((m-1)\)th and the
\(m\)th ray, respectively. The general expressions take
on the form

\[
r_m = 2r_{m-1} - r_{m-2},
\]

\[
\mu_m = \frac{\mu_m-1}{r_m \sin \left[ \phi_{m-1} + \sin^{-1}\left( \frac{r_{m-1}}{r_m} \sin j_{m-1} \right) \right] - \frac{1}{r_m}}.
\]

At the expense of simplicity, the ray-tracing in
version method can be generalized to include cases
where the refractive index within each layer is allowed
to change both in the vertical and in the horizontal
directions. However, when only a single occultation
is observed, one must specify the horizontal changes
before the data can be utilized to determine the vertical
refractive index profile. Instead of proceeding to a dis-
cussion of these more complex algorithms, we shall
devote the last appendix to a derivation of a simple
analytical solution for \(\mu(r_0)\).

**APPENDIX C: ABELIAN INTEGRAL INVERSION**

The purpose of this appendix is to derive an integral
transform pair relating \(\alpha(a)\) and \(\mu(r_0)\). The resulting
formulation may be regarded as a special case of a
mathematical problem which was solved by Abel in
1826.

Figure 23 defines the geometrical parameters needed
in the analysis. The angle of incidence \(i\) is related to
the polar ray-path coordinates \((r, \phi)\) by

\[
\tan i = \frac{r d\phi}{dr}.
\]

Another expression involving \(i\) may be obtained from
equations given in Appendix B. Multiplying Eqs. (B2),
(B3), (B5), and the corresponding equations for all the
other boundaries between the first and the \(n\)th layer yields

\[
a_m = \mu_n \sin i_m n.
\]

By letting the number of layers become infinitely
large, we can apply this formula to an atmosphere
where \(\mu\) is a continuous function of \(r\). For this limiting
case, we find

\[
a = \mu r \sin i,
\]

where the subscripts have been omitted. This equation
is known as Bouger's rule (Born and Wolf 1959). At the
point of closest approach on the ray path, we have

\[
a = \mu (r_0) r_0.
\]

The sum of the three angles \(\phi\), \(i\), and \(\psi\) defined in
Fig. 23 is equal to \(\pi/2\). The differentials of these
parameters are therefore related by

\[
d\phi = -di - d\psi.
\]

An expression for \(di\) may be obtained by differentiating
Eq. (C3). This operation gives

\[
di = -\frac{a(\mu + (d\mu/dr)r)dr}{\mu r [\mu r^2 - a^2]}.\]

Combining Eqs. (C1) and (C3) and eliminating \(i\) yields

\[
d\phi = \frac{a dr}{r [\mu r^2 - a^2]}.\]

Equations (C5)–(C7) allows us to relate \(d\phi\) to \(dr\) and \(r\):

\[
d\phi = \frac{a d\mu/dr}{[\mu r^2 - a^2] d\mu/\mu dr}.
\]

Integration of \(d\phi\) along the entire ray path yields the
angular deflection of the beam,

\[
\alpha(a) = 2a \int_{r=r_0}^{r=\infty} \frac{d\mu}{\mu dr} \left[ \frac{dr}{[\mu r^2 - a^2]} \right].
\]

Equation (C9) may be inverted by employing
standard mathematical techniques (Hamel 1937). First,
we introduce a dummy variable of integration \(x\) to
replace the product \(\mu r^2\),

\[
\alpha(a) = 2a \int_{x=x_0}^{x=\infty} \frac{d\mu}{\mu dx} \left[ \frac{dx}{(x^2 - a^2)} \right].
\]
Next, let \( a_1 \) denote the asymptote miss distance for a ray whose radius of closest approach is \( r_{o1} \). We now multiply both sides of Eq. (C10) with the kernel \( (a^2-a_1^2)^{-1} \) and integrate with respect to \( a \) from \( a_1 \) to infinity. This manipulation yields

\[
\int_{a=a_1}^{a=\infty} \frac{\alpha(a)}{a^2-a_1^2} \, da = \int_{a=a_1}^{a=\infty} \frac{2a}{(a^2-a_1^2)^{1/2}} \left[ \int_{a=a_1}^{a=\infty} \frac{1}{\mu} \, dx \left( \frac{x^2-a_1^2}{x^2-a_1^4} \right) \right] \, da = \int_{a=a_1}^{a=\infty} \frac{1}{\mu} \, dx \left[ \int_{a=a_1}^{a=\infty} \frac{1}{\mu} \, dx \left( \frac{x^2-a_1^2}{x^2-a_1^4} \right) \right] \, dx = \pi \int_{a=a_1}^{a=\infty} 1 \, dx = -\pi \ln \mu(r_{o1}).
\]

The term containing \( \alpha(a) \) may be integrated by parts. This operation yields the following formula:

\[
\mu(r_{o1}) = \exp \left[ \frac{1}{\pi} \int_{a=a_1}^{a=\infty} \ln \left( \frac{\alpha(a)}{a_1} + \left[ \frac{\alpha(a)}{a_1} \right]^2 - 1 \right) \, da \right].
\]

(C11)

The last equation may be used to compute the refractive index of the atmosphere at a distance \( r_{o1} \) from the planetary center of mass. Here \( r_{o1} \) is given by

\[
r_{o1} = a_1/\mu(r_{o1}).
\]

(C12)

Due to refraction in the ionosphere, the integrand of Eq. (C11) is a multivalued function of \( \alpha \). It may therefore be easier to visualize integration with respect to \( \alpha \). Rearrangement of Eq. (C11) gives

\[
\mu(r_{o1}) = \exp \left[ \frac{1}{\pi} \int_{a=a_1}^{a=\infty} \ln \left( \frac{\alpha}{a_1} + \left[ \frac{\alpha}{a_1} \right]^2 - 1 \right) \, da \right].
\]

(C13)

The integrand of this equation is a single-valued function of \( \alpha \) when the radio frequency is high enough to avoid critical refraction on the bottom side of the ionosphere. (Critical refraction occurs when the radius of curvature of a horizontal ray is equal to the distance to the planetary center of mass.) The inversion methods described in this report were developed in the course of a long-range study of bistatic radar techniques for the exploration of the solar system.

For other discussions of inversion algorithms, the reader is referred to Fischbach (1965), Fjeldbo and Eshleman (1968), and Phinney and Anderson (1968). The first author outlines an interesting method for probing the terrestrial atmosphere by satellite measurements of stellar refraction. The second report describes techniques that may be used to analyze radio occultation data for the case of small ray bending. The last reference discusses analogies between radio occultation experiments and earlier studies of seismic wave propagation.

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