Tracing slant paths through structured grids

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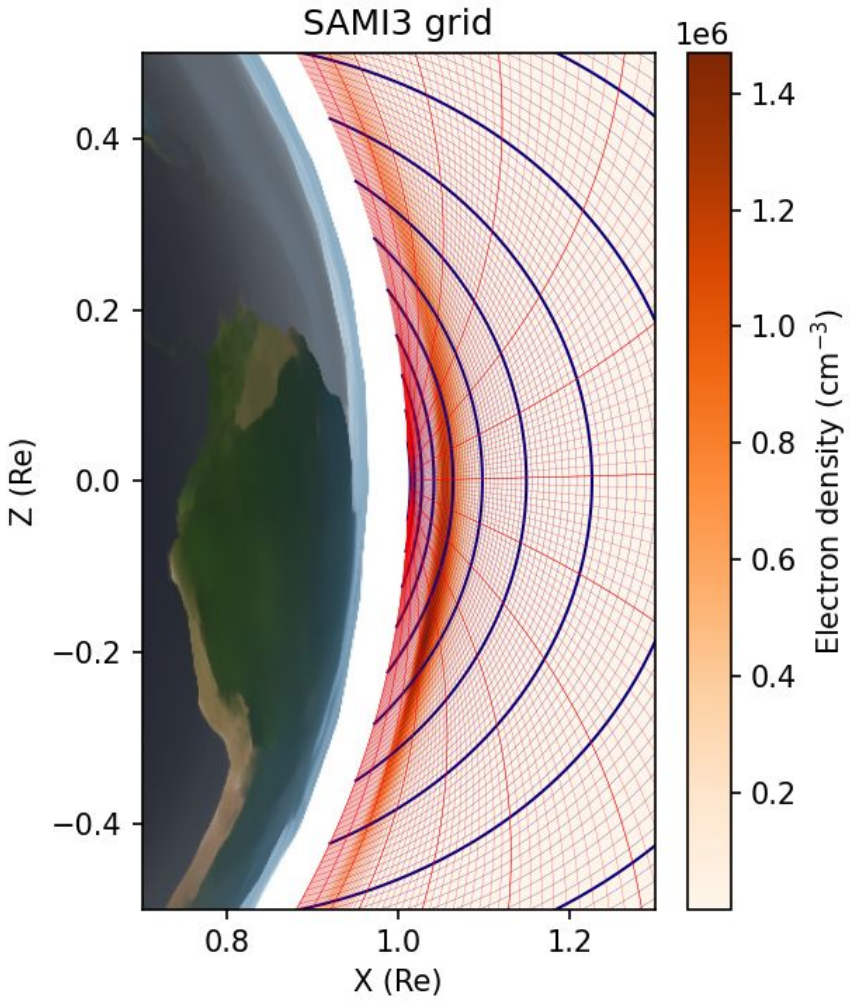
Overview/motivation

- Some grid geometries cannot be treated as a stack of 2-D grids at different altitudes
- Many observation operators require interpolating along a slant path through the model domain (e.g. TEC, GNSS-RO)
- Tracing through the grid along the observation line of sight provides an efficient means to perform the interpolation required for these cases



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GNSS observations

Dual-frequency GNSS signals can be used to estimate **line-integrated electron density** along the ray path from transmitter to receiver (called total electron content or TEC)

Applies to ground-based receivers and also satellite-based (e.g. radio occultation)

Direct processing requires integral along a slant path; transforms can be used to estimate vertical integrals or profiles from the raw data

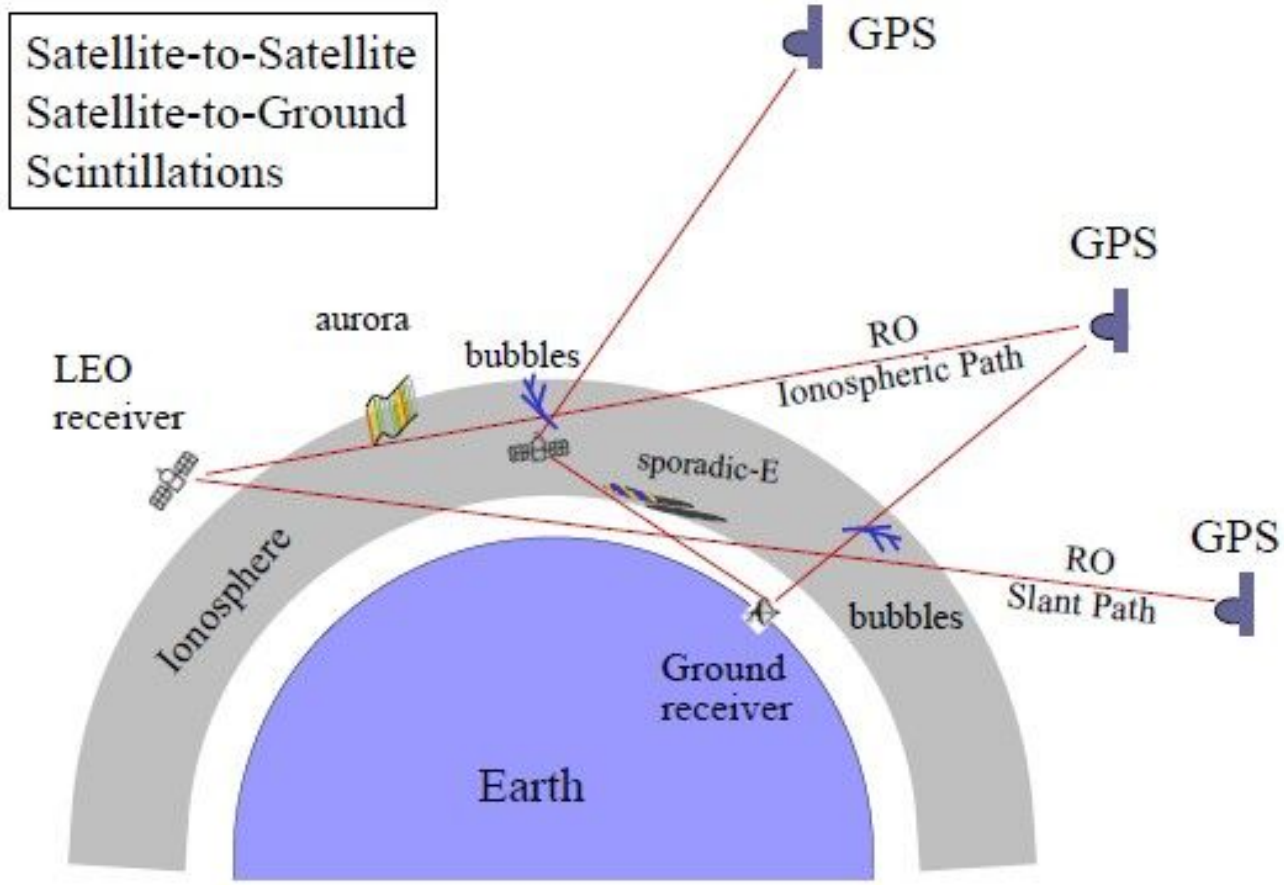

Illustration of GNSS ray paths through the ionosphere (Wu 2020)

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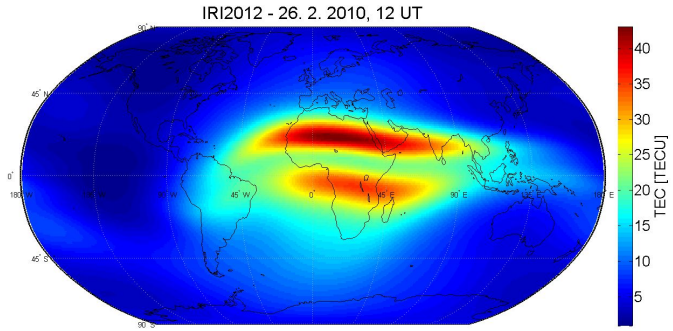
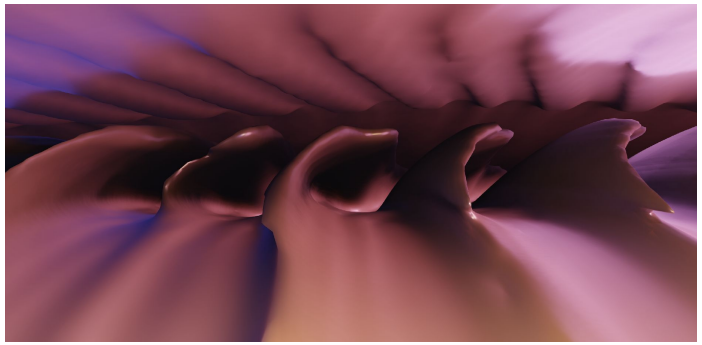
Model descriptions

Model	Description
SAMI3: SAMI3 is Another Model of the Ionosphere (Navy non-releasable code)	**Physics-based ionosphere model** that solves the Euler equations for multiple ion species with ionosphere-specific forcing terms
PyIRI: Python IRI (Publicly Available)	**Python re-write of IRI** (much faster than IRI)
IRI: International Reference Ionosphere (Publicly Available)	Empirical (statistical) model that provides **climatological average electron densities** given a date, time, and solar flux parameter



Global total electron density (TEC) from IRI (Najman+2014)



Electron density isosurfaces from a SAMI3 simulation

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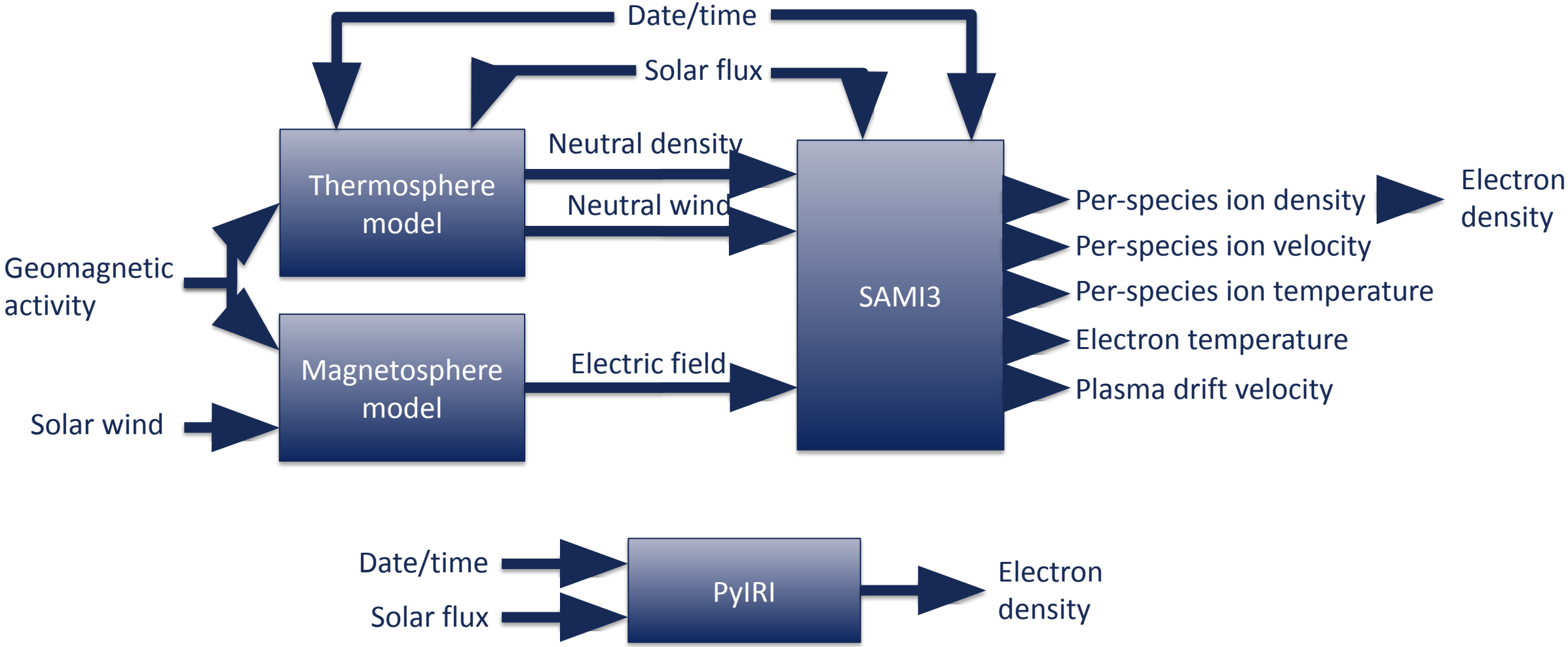
Model inputs/outputs



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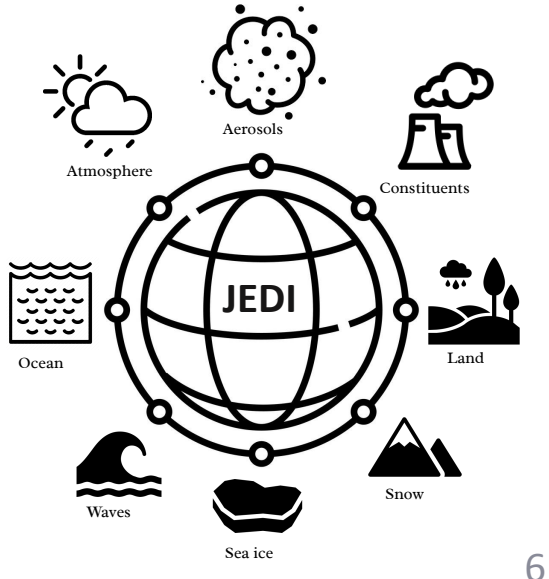
PyIRI-JEDI

- Serves as a testbed for using JEDI with SAMI3
- **Publicly-available source code** of PyIRI **maximizes collaboration**
- Wrapper around PyIRI enables it to use **same JEDI interface as SAMI3**
  - Output interpolated onto a geomagnetic grid
  - Electron density divided across ion species
  - Output written in file formats resembling those used by SAMI3



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Differences from typical atmospheric models

	Terrestrial weather models	SAMI3
Coordinate system	**Geographic** (latitude, longitude, altitude)	**Geomagnetic** (magnetic latitude, magnetic longitude, altitude)
Grids	Varied, but **vertical columns over a spherical shell** are common.	Grid follows magnetic field lines. Grid points **do not align in vertical columns**.
System of equations	Navier-Stokes, possibly with chemistry	Euler equations plus a subset of Maxwell's Equations, with ion chemistry
State variables	Neutral densities, velocities, temperatures	Per-species ion densities, per-species temperatures, per-species velocities

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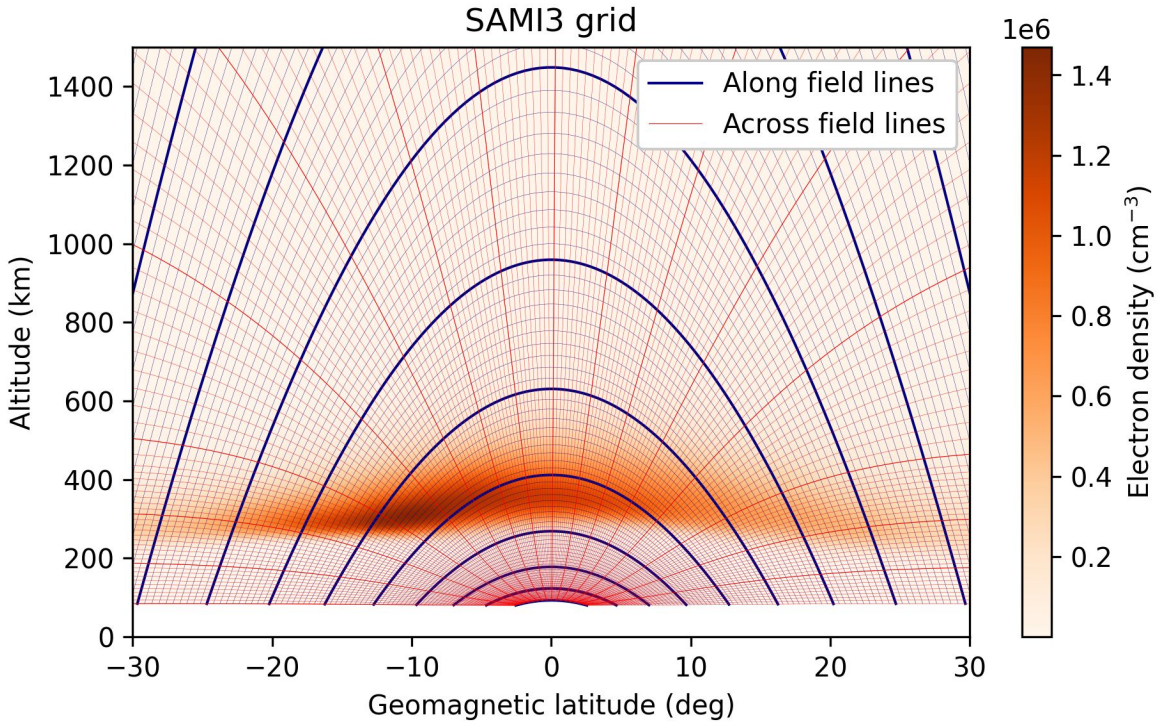
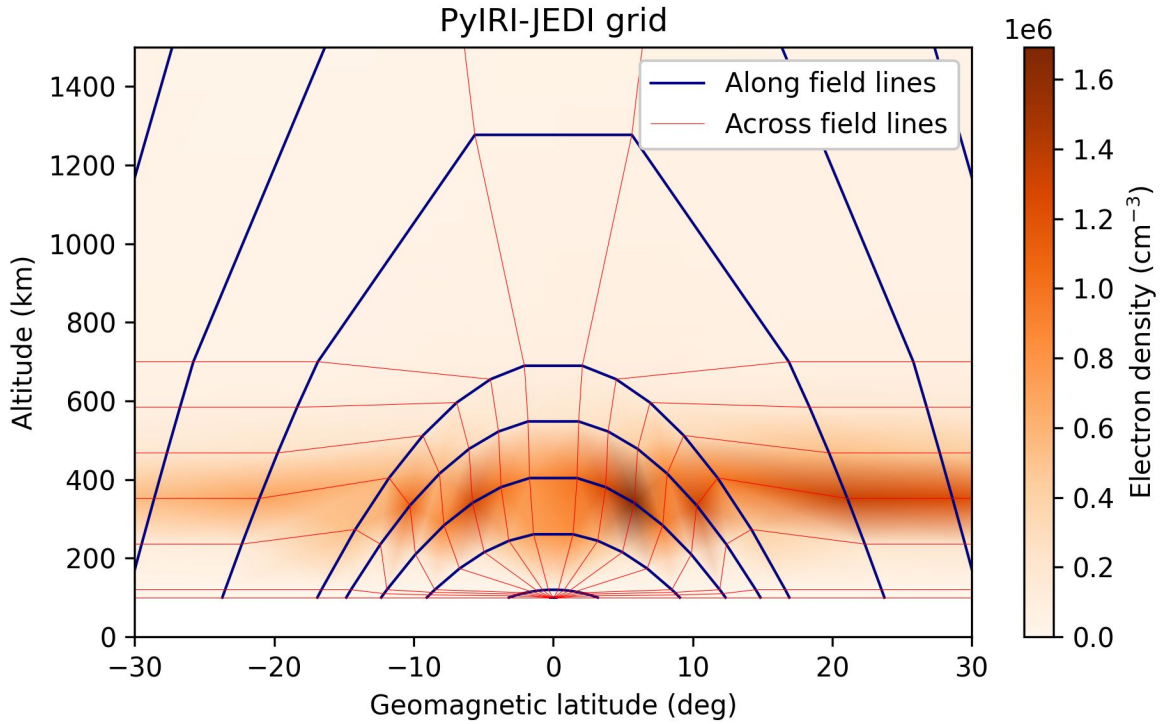
Geomagnetic grid

- Grid **aligns with magnetic field**
- Native coordinates are geomagnetic
- Grid points concentrated at ionospheric altitudes





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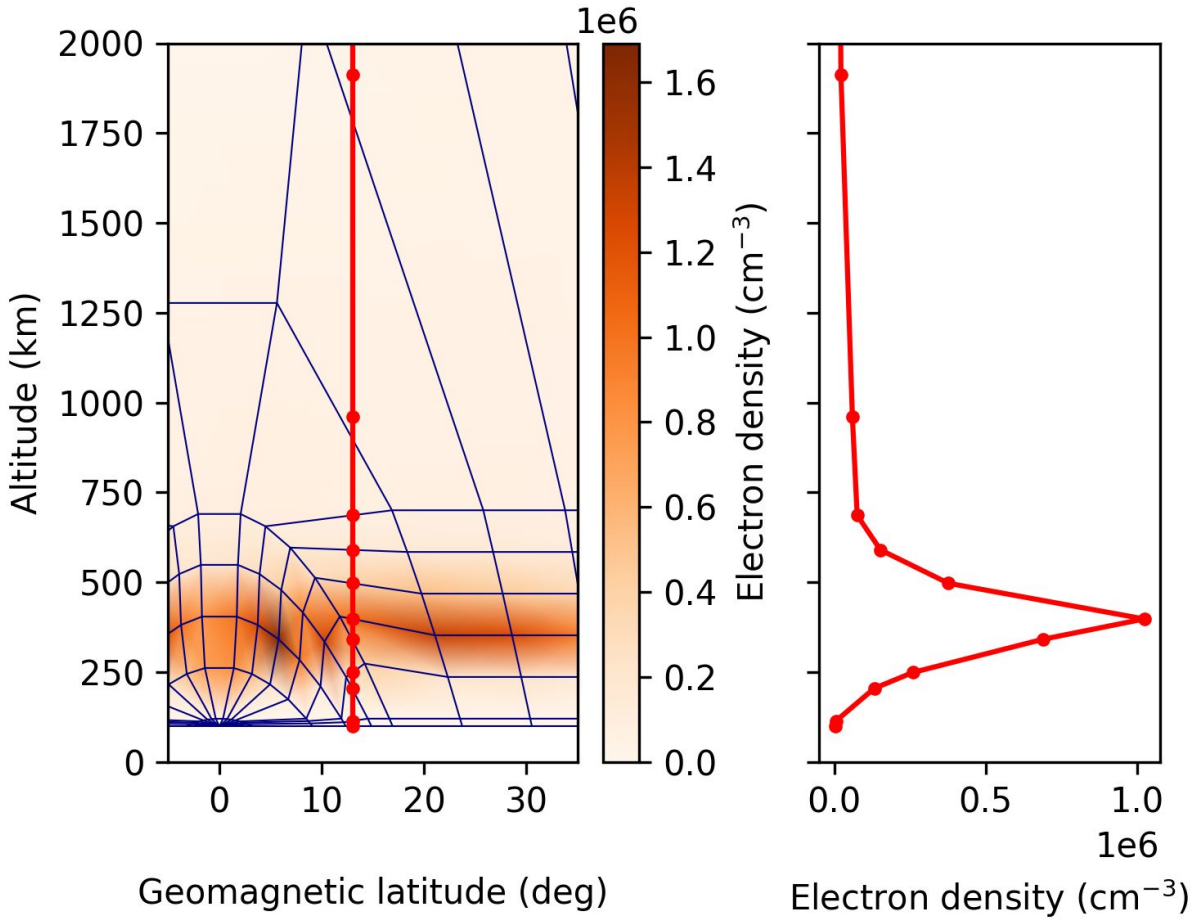
Grid interpolation

- JEDI observation operators operate on vertical columns
- Grid complexities not found in existing JEDI models:
  - **Grid points not in vertical columns** (requires custom interpolator code)
  - **Number of points varies with latitude** (addressed by storing interpolator output in an oversized buffer with fill values)
  - Latitude/longitudes in geomagnetic (not geographic) coordinates



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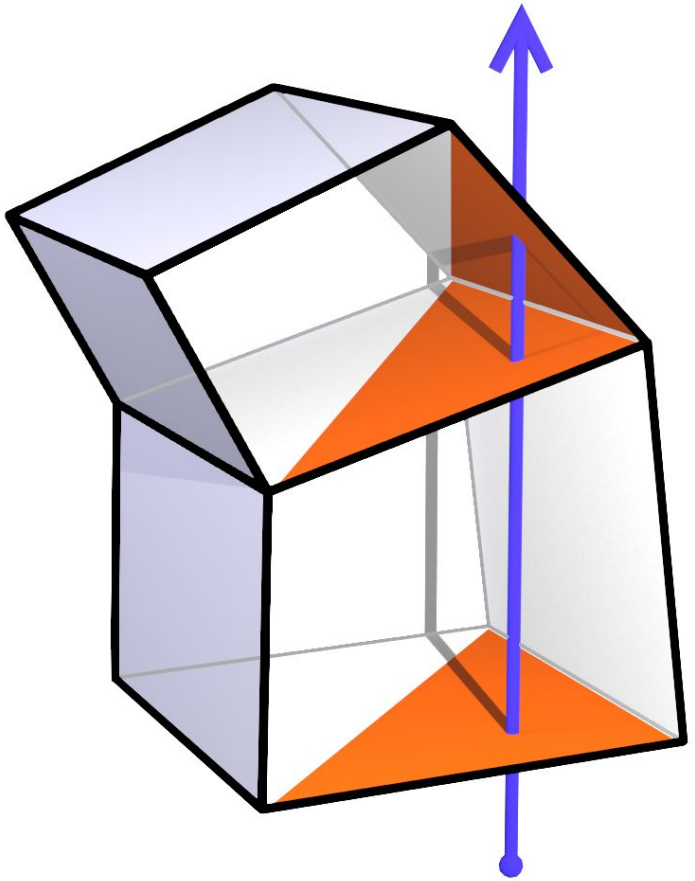
Ray-tracing based interpolation

- Locate where ray intersects cell faces
- Interpolate within each face at the intersection point
- Implementation requires a structured grid of hexahedral cells
- No assumptions made about cell/vertex locations
- Agnostic to ray direction
- Currently, only straight rays are implemented (i.e. no refraction)



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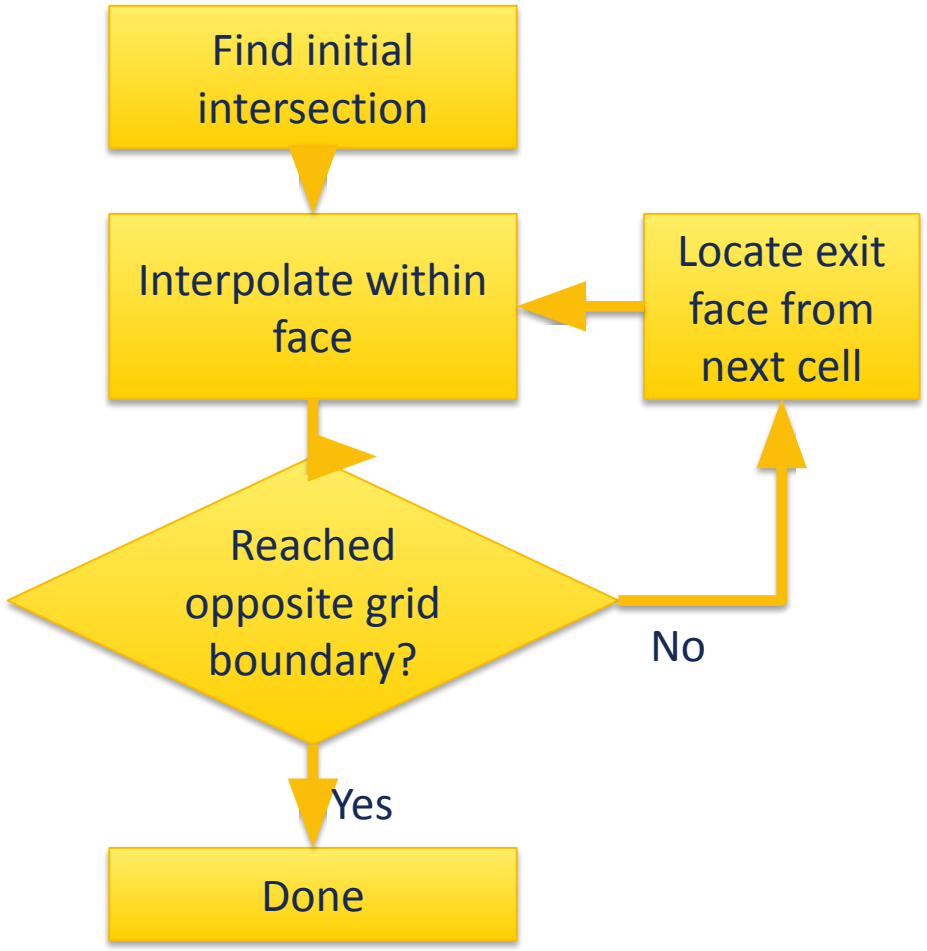
Ray-tracing based interpolation



- Once initial intersection is found, complexity is O(n) where n is the number of points in the column
- Finding initial intersection is $O(2*(n_i*n_j+n_i*n_k+n_j*n_k))$ but only has to be done once per ray (and could be sped up with a K-D tree or similar)

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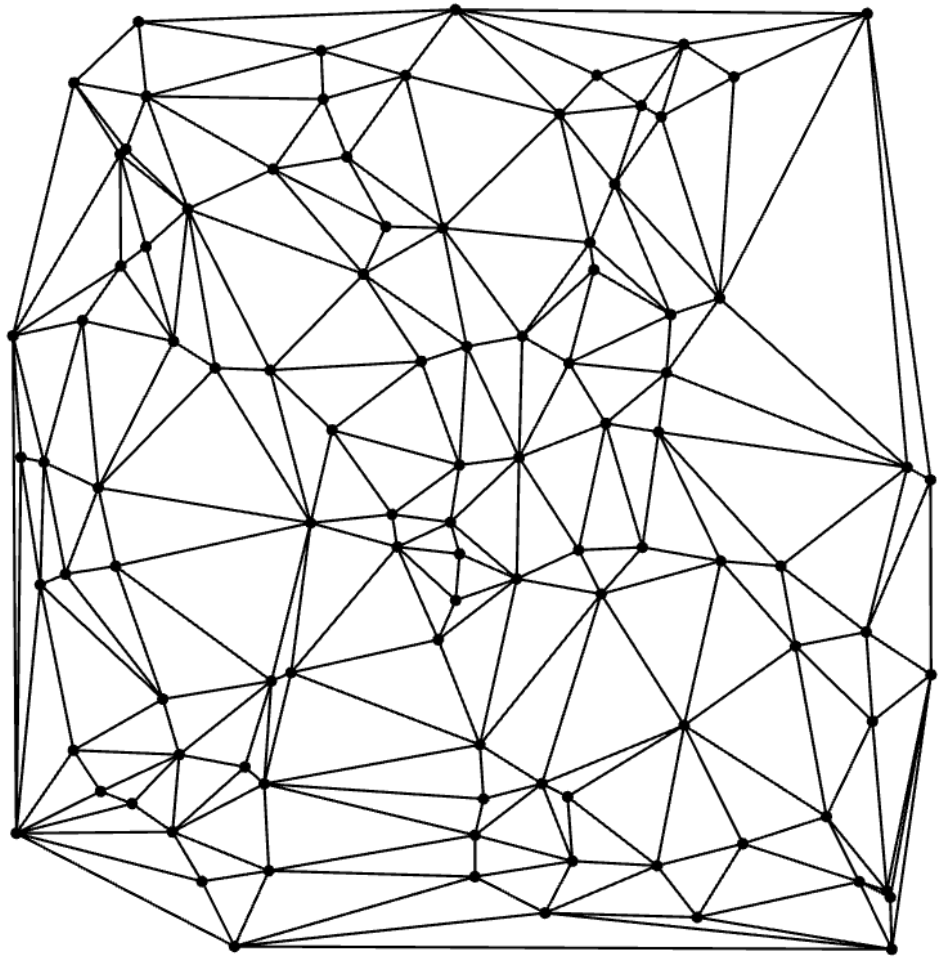
Delaunay triangulation

- Treats grid as a point cloud
- Connects grid points using triangles (tetrahedra in 3D case)
- Computational complexity is $O(n_p * \log(n_p))$ where n_p is the number of points in the grid
- Locating an arbitrary point within the triangulation can cost up to $O(n_t)$ where n_t is the number of triangles



Example of a 2D Delaunay triangulation (Wikipedia)

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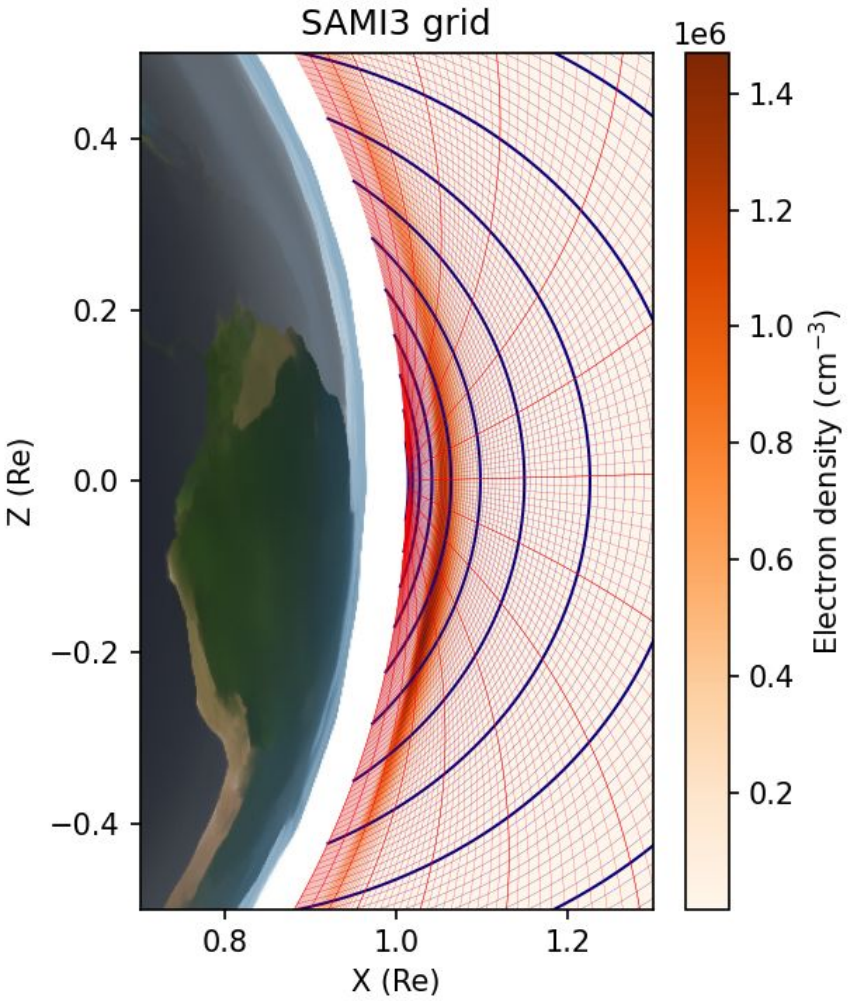
Reducing dimensionality

- When viewed in geomagnetic coordinates, SAMI3 grid looks like a 2-D grid that was extruded in longitude
- Locating a point in longitude space is trivial (linear formula)
- Ray tracing can be performed in 2-D space, significantly reducing the computational cost
- Only works when the ray stays within a single longitude (i.e. vertical rays)
- Geographic input coordinates must be transformed to geomagnetic, and errors in this transform result in an incorrect interpolation



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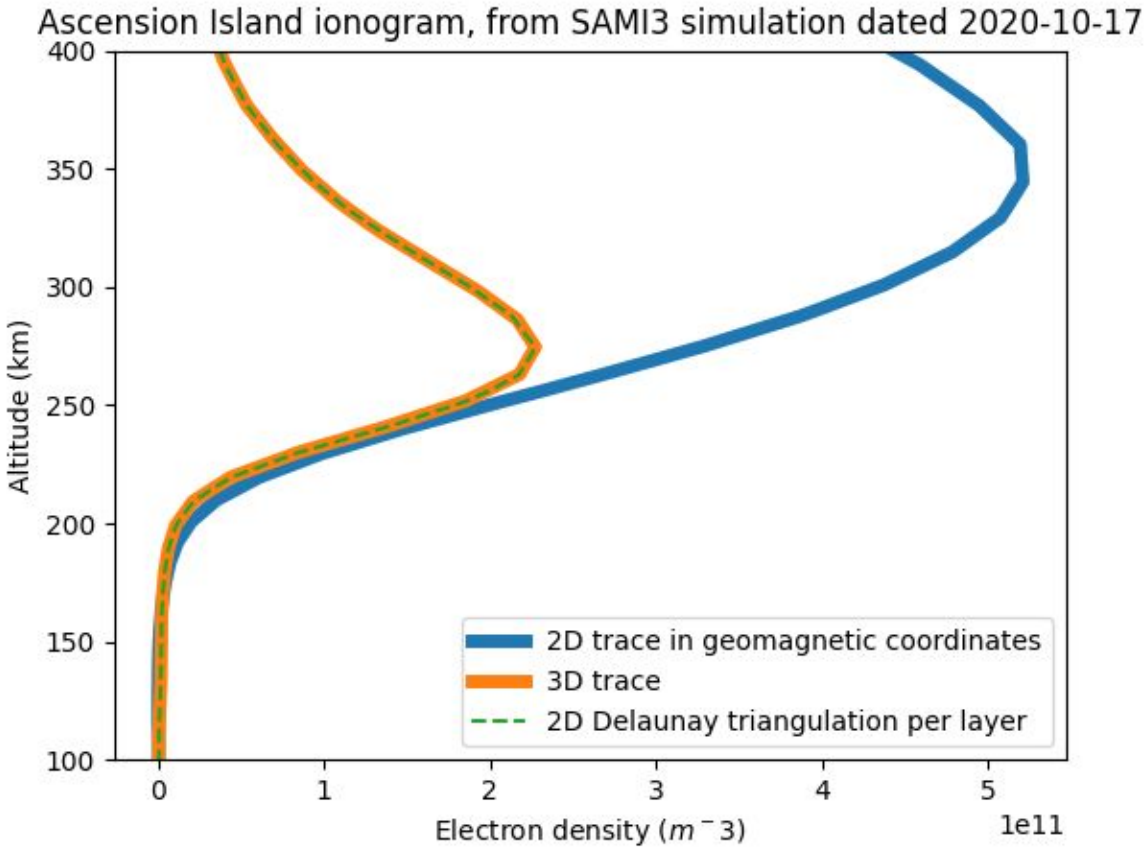
Speed comparison

Interpolation scheme	Time to trace one profile (s)
2D trace	0.0024
3D trace	0.0254
2D Delaunay triangulation at each level	20.42
LinearNDInterpolator (3D Delaunay triangulation)	16.39*



*Plus 3 minutes to compute Delaunay triangulation, or 1-6 seconds to read a pre-computed triangulation from disk.

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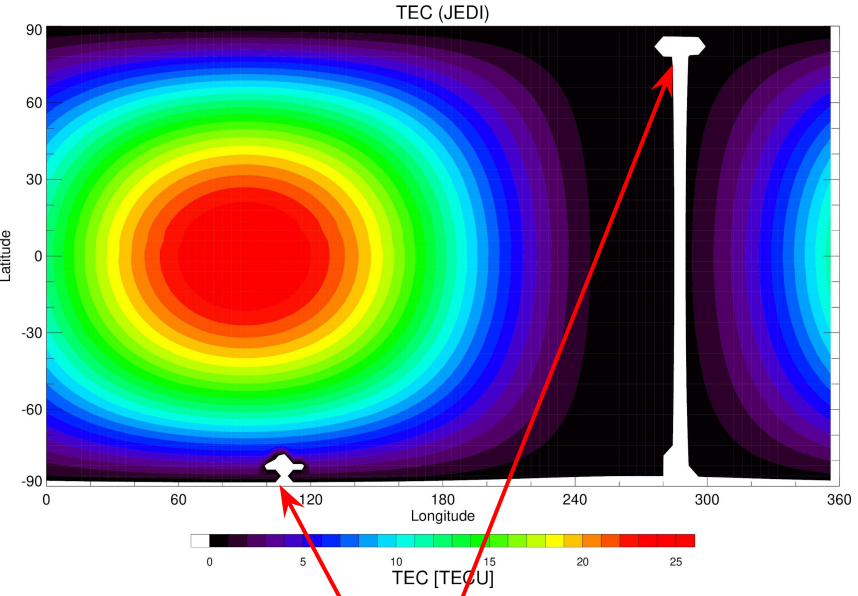
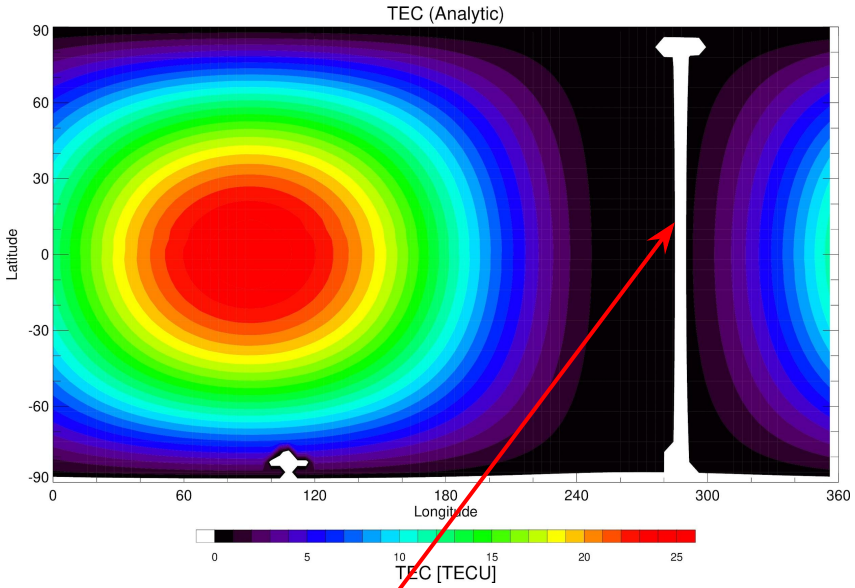
Accuracy

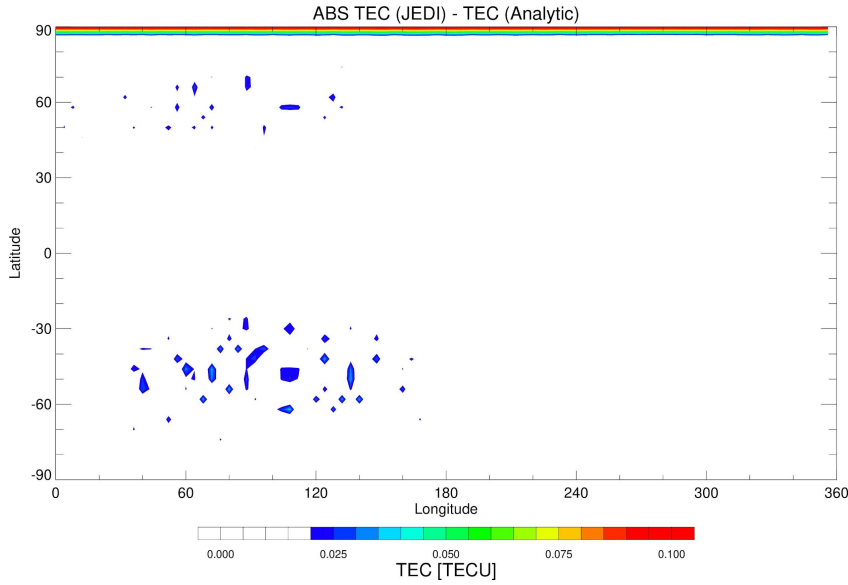
Structured grid ray tracer with Cartesian coordinates



Analytic (truth) values



Differences



Magnetic poles

Magnetic “prime meridian”

White areas on the plots are regions not covered by the grid

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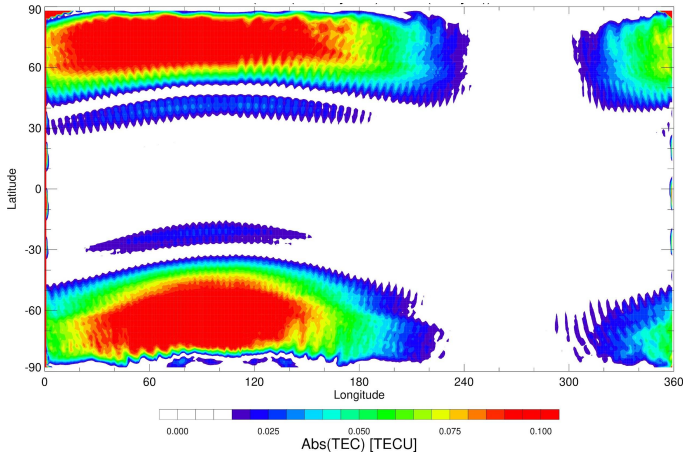
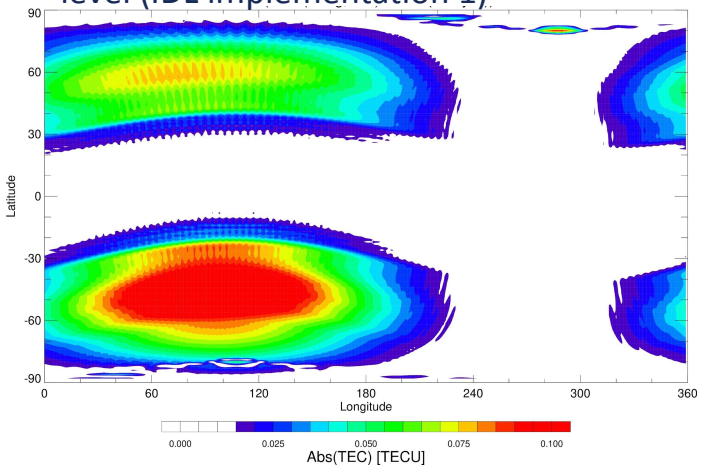
Accuracy

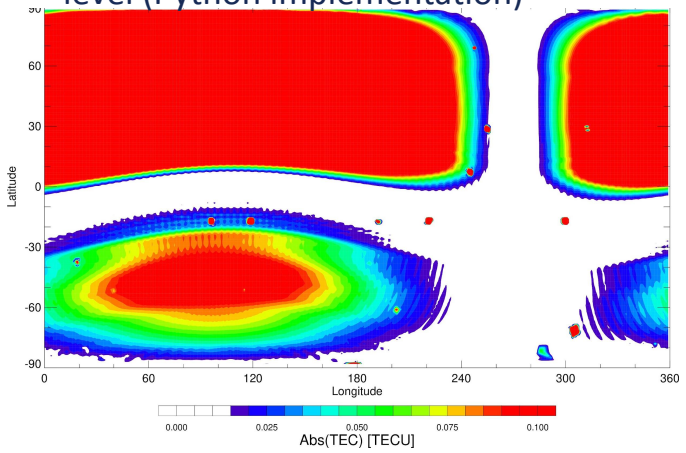
3D Delaunay triangulation



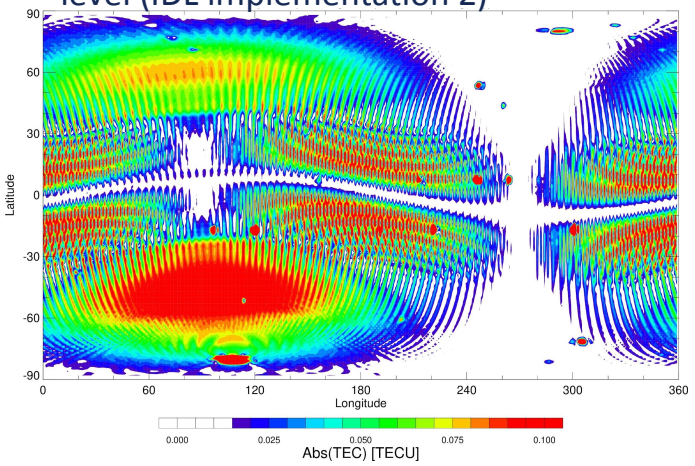
2D Delaunay triangulation per level (IDL implementation 1)



2D Delaunay triangulation per level (Python implementation)

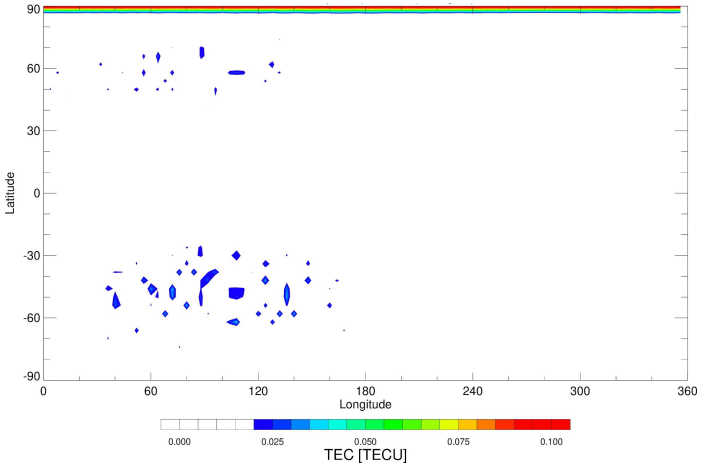


2D Delaunay triangulation per level (IDL implementation 2)



abs(Interp – Truth)

3D trace in geographic coordinates



Interpolator	RMS (area-weighted)
2D Delaunay, IDL 1	0.0369689
2D Delaunay, IDL 2	0.0645227
2D Delaunay, Python	0.3307453
3D Delaunay	0.0329854
3D trace	0.00723197

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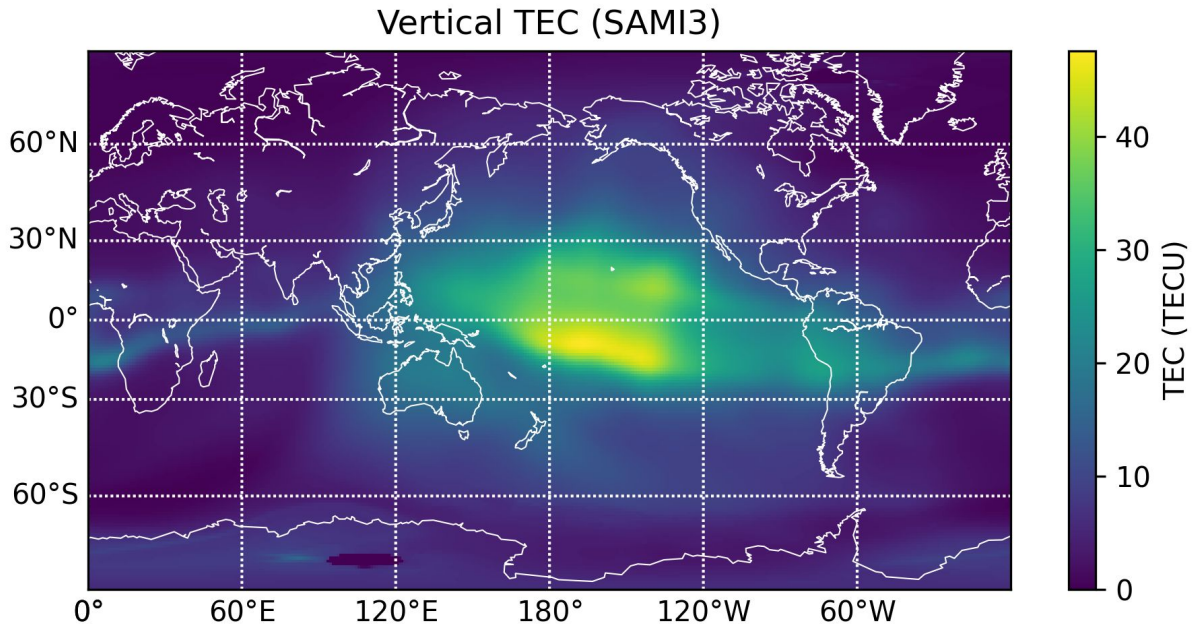
Observation operators

- Two observation operators implemented so far:
  - **Generic point observation** operator that returns a state variable at a given latitude, longitude, and altitude
  - **Vertical total electron content** (TEC) operator that gives the integral of electron density above a given latitude and longitude



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Summary

- Ray tracing interpolator appears to be faster and more accurate than alternatives available to handle the SAMI3 grid
- Next steps:
  - Interface interpolator to UFO
  - Test slant path interpolation
  - Investigate ways to support our grid geometry within atlas